CHAPTER 3 FUZZY LOGIC CONTROL FOR INVERTED PENDULUM SYSTEM

An inverted pendulum is generally a simple mechanical arrangement of a rigid pole hinged to a moving rail-cart system. Formulated by Minorsky [90] this system has enormous application in various industries. It forms a basic component of robotics industry, a single link manipulator where the link is controlled with the help of the joint motor. An overhead crane arrangement used at shipping or material handling industries is extension of simple cart-pole arrangement, wherein control objective is to reduce the oscillations of the object being transferred. A missile system is advanced illustration of cart-pole concept, although it doesn't exhibit the exact similarity to a cart-pole system but has similar dynamics as of pole-balancing problem [91].

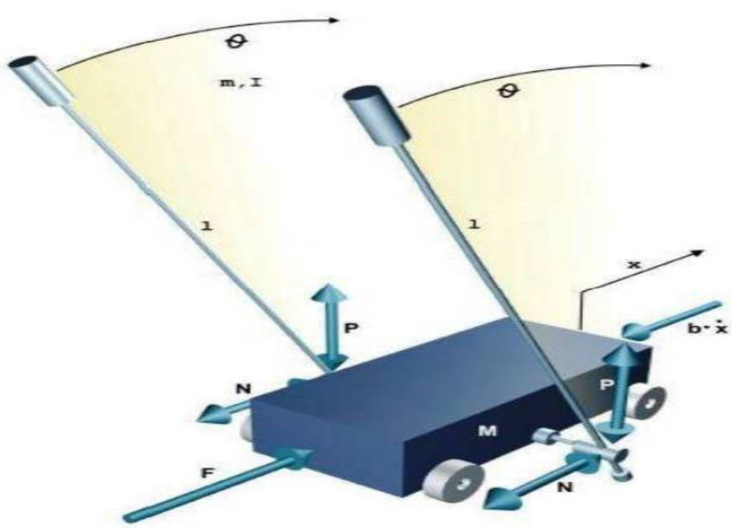
This chapter discusses the implementation of proposed optimized controller for control of a twin arm digital pendulum system. The performance of the proposed controller is compared to the performance obtained for PID and fuzzy logic control on based on the following parameters:

- Transient response parameter: peak overshoot.
- Steady – state parameters: settling time, steady – state error.
- Performance indices.

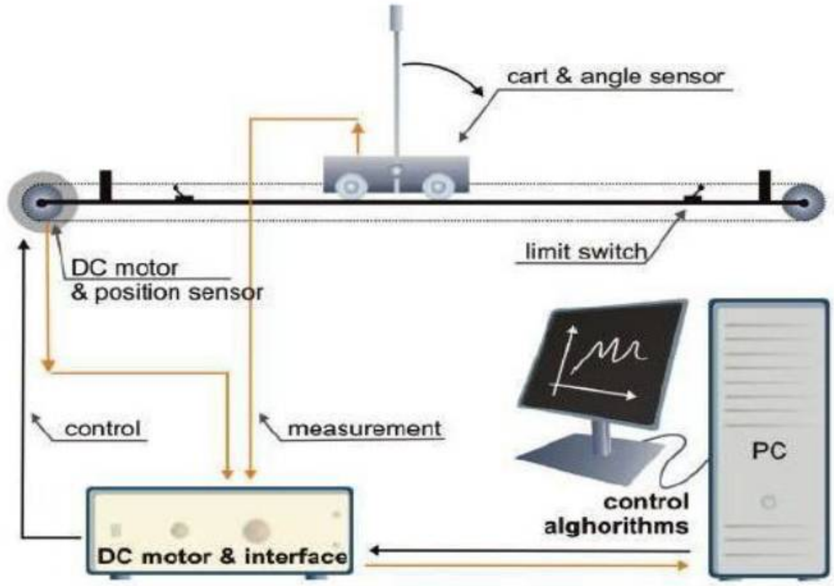
3. 1 Problem formulation

Control of Inverted Pendulum is a benchmark non-linear control problem. The control philosophy for inverted pendulum is fairly simple: here the controller's job is: swing upright and maintain the position of pendulum to inverted position by counteracting the earth's gravitational force. To generate counteracting force the cart is moved back and forth, due to which the pendulum gains inertia leading to an oscillatory motion. Once the pendulum reaches the desired inverted position the cart tried to maintain the inverted position. However the challenge is because of the inherent instability of under-actuated pendulum system [92]. Being a non-linear system the response of classis PID controller is governed by the linearization of the system dynamics resulting in a limited operating range. However to design an effective FLCs we require expert system knowledge, and is demonstrated in numerous researches and is fairly simple. With this perspective we design a FLC that moves the cart back and forth as per the pendulums desired position and tries to preserve the pendulum balance as it reaches the desired inverted position. The current study exploits the swing-up control for inverted pendulum where it is brought to an inverted position from its natural equilibrium($\theta = 180°$).

Figure 3-1 (a) depicts the forces acting on a twin arm inverted pendulum consisting of a cart, rigidly coupled twin pendulums to a cart which moves back and forth on a rail. The pendulum arms are rigidly hinged to the cart's center and rotate around its pivot point. Figure 3-1 (b) depicts the control layout of computer control for digital pendulum control system (DPCS). Figure also shows the physical setup and connections between computer and DAQ card. The DAQ card acts as a real-time interface device between the analog cart-pendulum system and the digital computer. Also the signals generated form analog position sensors via cart and pendulum movements are converted to digital from, similarly the digital control signal generated by PC via Matlab-Simulink™ are converted to analog form via the DAQ card.



(a)



(b)

Figure 3-1 (a) Forces acting on pendulum system and (b) Computer control for real-time DPCS **[93]**

3. 2 Mathematical modeling

Balancing the forces acting on cart-pendulum system depicted in Figure 3-1 (a). The mathematically model of the system is written as:

$$F = (m + M)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta \quad (3\text{-}1)$$

$$(I + ml^2)\ddot{\theta} - mglsin\theta + ml\ddot{x}cos\theta + d\dot{\theta} = 0 \quad (3\text{-}2)$$

Classical control theory is designed around linear systems. Therefore to design a PID controller linear model of the pendulum system is required. As the linearization is done around the operating point here, the equations are thus linearized by assuming$\theta = 0$. Here we get:

$$F = (m + M)\ddot{x} + b\dot{x} - ml\ddot{\theta} \quad (3\text{-}3)$$

$$(I + ml^2)\ddot{\theta} + mgl\theta - ml\ddot{x} + d\dot{\theta} = 0 \quad (3\text{-}4)$$

Figure 3-2 depicts the real-time model available in control theory and simulation lab. Table 3-1 depicts the system parameters for the real-time hardware model. The model utilized for the experimental purpose is: “Feedback instruments™ digital pendulum system: 33-936S” [93].

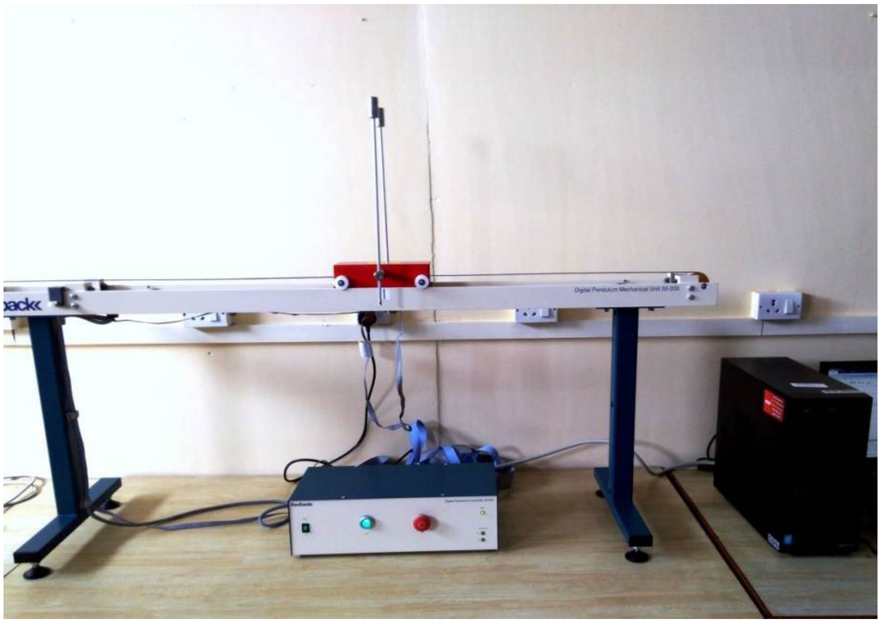


Figure 3-2 Real-time DPCS system available in Control theory and simulation lab, UPES

Table 3-1 Parameters for real-time model

Parameter	Description	Value
M	Mass of cart	2.4 Kg
m	Mass of pendulum (combined mass for twin pendulums)	0.23 Kg
l	length of pendulum	0.4 m
g	acceleration due to gravity	9.81 m/s^2
F	force applied on the cart by motor	Variable
x	The distance moved by cart	Variable
b	friction coefficient between rail and cart	0.05 Ns/m
θ	pendulum angular position (w.r.t positive y–axis considered as 0^o angle)	Variable
$\dot{\theta}$	Pendulum angular velocity	Variable
$\ddot{\theta}$	pendulum angular acceleration	Variable

3. 3 Fuzzy logic control

An inherent benefit of a fuzzy logic based system is that, the FLS is designed independent of system dynamics and are designed utilizing the expert knowledge base. However tuning of PID controller requires system dynamics and is designed around system’s linear model. Hence, mathematical modeling is utilized for obtaining PID controller gains and the FLC is designed by treating the pendulum system as a black-box system overlooking the system dynamics, considering the expert knowledge for pendulum system. The FLC architecture for DPCS system is portrayed in Figure 3-3.

The controller used here uses Mamdani architecture and is of PD type, consequently error in pendulum angle its differential is fed-back to the controller. The error is: "the difference between desired angular position and the measured angular position". The desired inverted position of the pendulum is termed as $0°$ angle, with initial conditions being the natural equilibrium position of a simple pendulum i.e. angle $=180°$. The pendulum is now swinged up to an inverted position with subsequent cart movements and pendulum angle is now maintained at an inverted position.

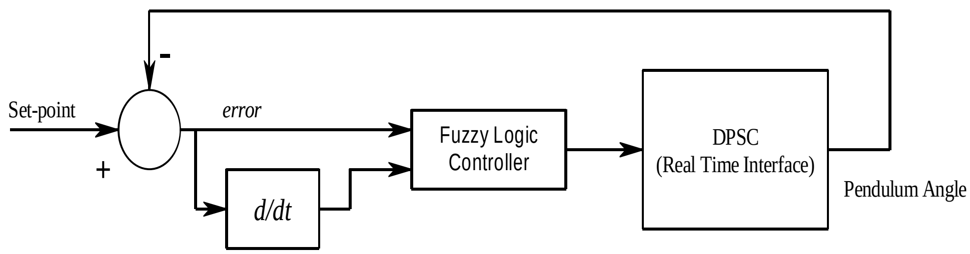


Figure 3-3 Fuzzy logic control for DPCS

The initial sets designed for error in pendulum angle can be seen in Figure 3-4. The sets are designed according to the procedure discussed in section 2. 4, here the fuzzy sets are named according to position with zero error.

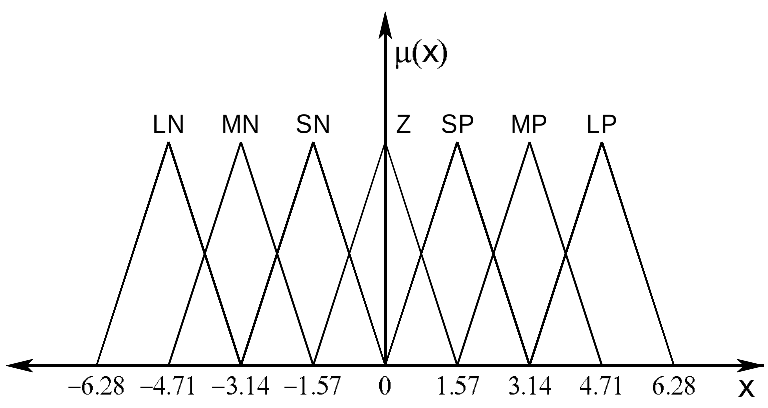


Figure 3-4 FS for error in pendulum angular position

FS for "$\dot{e}$ – rate of change of error" & "c – control force" are designed using similar nomenclature. The range for these sets is:

$$\dot{e} = [-14.3, 14.3] \qquad c = [-5, 5]$$

The rule base for error, rate of change of error vs control force is summarized in Table 3-2.

Table 3-2 Rule base for pendulum angle controller using Mamdani FIS

Control force to DC motor		Rate of change of error ($\dot{e}$)						
		LN	MN	SN	Z	SP	MP	LP
Error in angle (e)	LN	LN	LN	LN	LN	MN	SN	Z
	MN	LN	LN	LN	MN	SN	Z	SP
	SN	LN	LN	MN	SN	Z	SP	MP
	Z	LN	MN	SN	Z	SP	MP	LP
	SP	MN	SN	Z	SP	MP	LP	LP
	MP	SN	Z	SP	MP	LP	LP	LP
	LP	Z	SP	MP	LP	LP	LP	LP

The rules derived tom the pendulum balancing principle discussed in section 3. 1; "the cart is moved back and forth to maintain pendulum angle to an inverted position". An example from Table 3-2 demonstrating the principle is:

If error is 'small negative' and change in error is 'small negative' than the control force should be 'medium negative'.

3. 3. 1 Optimized fuzzy logic control

To compute the optimized support displaced FSs are utilized. Figure 3-5 depicts displaced FS "Zero". Considering data obtained for error in angular position we proceed to find optimized set for "error in angle", "rate of change of error" and "control force". Standard deviation (σ) calculated are:

$$\sigma_e = 1.6752 \qquad \sigma_{\dot{e}} = 3.57 \qquad \sigma_c = 0.3248$$

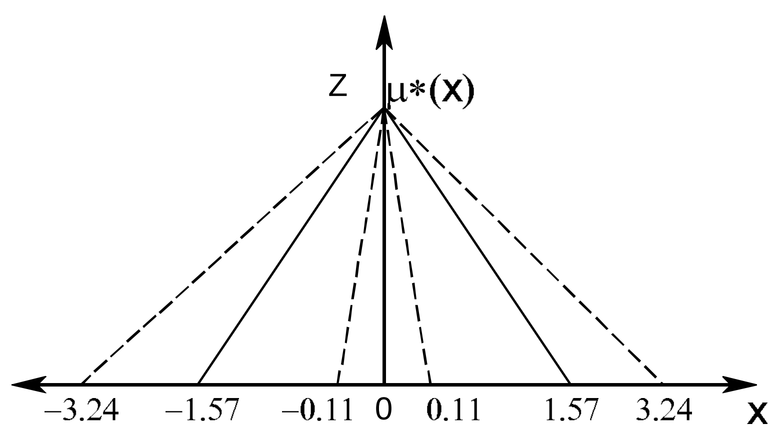


Figure 3-5 Displaced FS "Zero"

The next step in obtaining optimized support is: optimize the defined objective function and compute the optimal support for predefined FS. The objective function for FS "Zero" is:

$$H(\mu_{z^*}) = -\left[\int_{-\varepsilon\mp\sigma}^{0}\left(\frac{x+\varepsilon\mp\sigma}{\varepsilon\mp\sigma}\right)ln\left(\frac{x+\varepsilon\mp\sigma}{\varepsilon\mp\sigma}\right)dx + \int_{-\varepsilon\mp\sigma}^{0}\left(-\frac{x}{\varepsilon\mp\varepsilon}\right)ln\left(-\frac{x}{\varepsilon\mp\varepsilon}\right)dx\right] \\ -\left[\int_{0}^{\varepsilon\pm\sigma}\left(\frac{\varepsilon\pm\sigma-x}{\varepsilon\pm\sigma}\right)ln\left(\frac{\varepsilon\pm\sigma-x}{\varepsilon\pm\sigma}\right)dx + \int_{0}^{\varepsilon\pm\sigma}\left(\frac{x}{\varepsilon\pm\sigma}\right)ln\left(\frac{x}{\varepsilon\pm\sigma}\right)dx\right] \tag{3-5}$$

3. 4 Simulation model and real-time experiment results

Figure 3-6 depicts the simulation model for the pendulum system and parameters (for the linearized system) for simulation model are depicted in Figure 3-7. The first step for designing simulation model is to obtain linear model of pendulum system for PID controller design. Controller is provided with two inputs: error in pendulum angle and cart position. The error in pendulum angle is calculates as follows:

$$e = \theta_{sp} - \theta_m \tag{3-6}$$

where θ_{sp} is the desired pendulum angle ($\theta_{sp} = 0$) for inverted pendulum position, and θ_m is the measured pendulum angle.

The controller output is: "control voltage" which is supplied to cart motor which moves the cart in either direction so as to swing the pendulum.

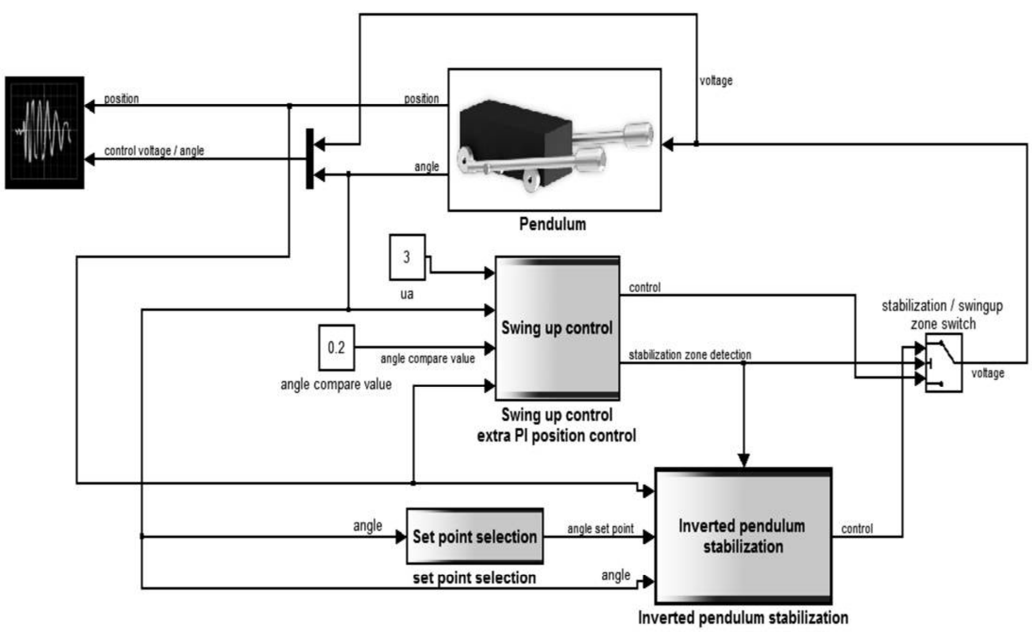


Figure 3-6 Simulation model for inverted pendulum system

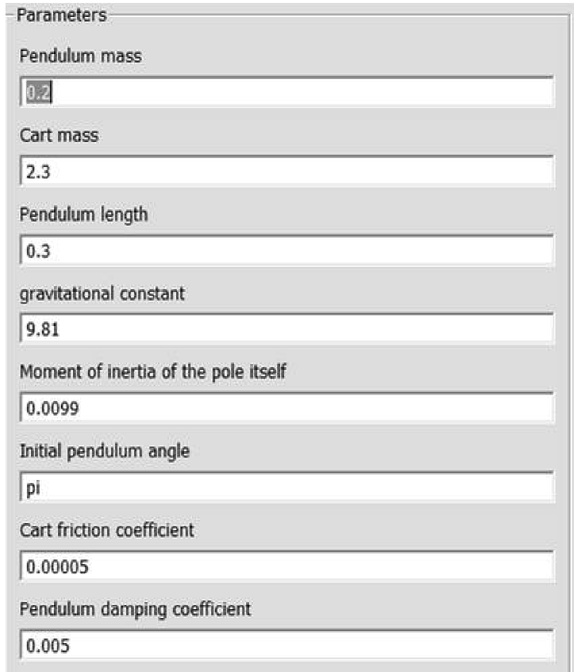


Figure 3-7 Parameters for simulation model

The results depicted here are acquired from real-time hardware model depicted in Figure 3-2. Since the PID controller is designed for the linearized model and is then implemented for non-linear practical model. The PID gains are obtained using the optimized Zeigler-Nichols (ZN) tuning method. For this "initial" PID gains are obtained using ZN tuning and are then optimized for minimum error-indices. This is evaluated using the simulation model for pendulum system. The simulation model objective is to maintain the pendulum angle at a desired position (i.e.$\theta_m = 0$). For this PID controller generates a control voltage which moves the cart such that the pendulum is swinged to achieve an inverted position and maintained at an inverted position thereafter.

3. 4. 1 Real-time PID control

Being a universal tool for control system performance evaluation PID control is benchmark to evaluate the performance for proposed controller. First we analyze PID control implementation for the digital pendulum system. Figure 3-8 shows the PID control architecture for real time pendulum angle control.

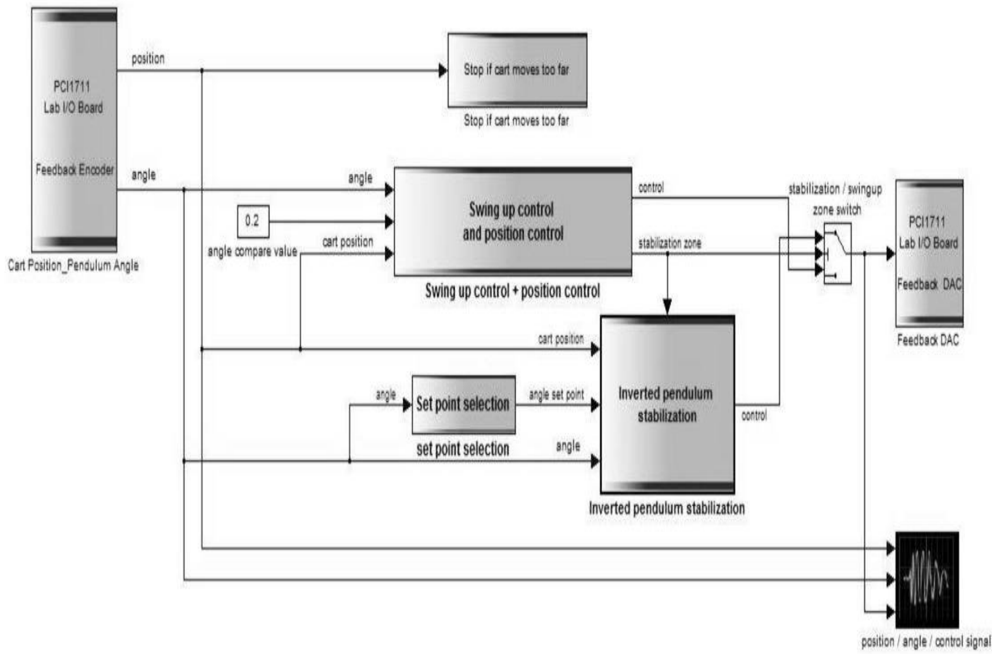


Figure 3-8 Real-time PID control for DPCS **[93]**

The controller response for default parameters is given in Figure 3-9. This figure shows the response obtained for pendulum angle with PID controller, where initial pendulum angle is 180°. The pendulum is brought to inverted position with help of back and forth cart movements, wherein Figure 3-10 shows the cart position with respect to time. Figure 3-11 shows the control force generated by PID controller.

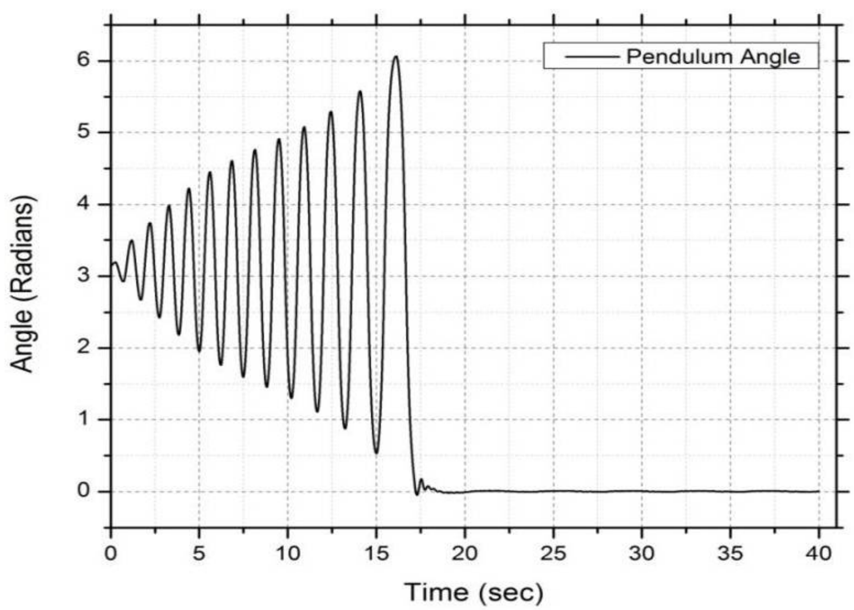


Figure 3-9 Pendulum angle for PID controller

As we implement the swing up stabilization for inverted pendulum the initial pendulum position is taken as its natural equilibrium from here pendulum is swinged to reach its desired position. To bring pendulum to inverted position the PID controller moves the cart back and forth, illustrated below in Figure 3-10 & swings the pendulum and uses its inertia to achieve inverted position.

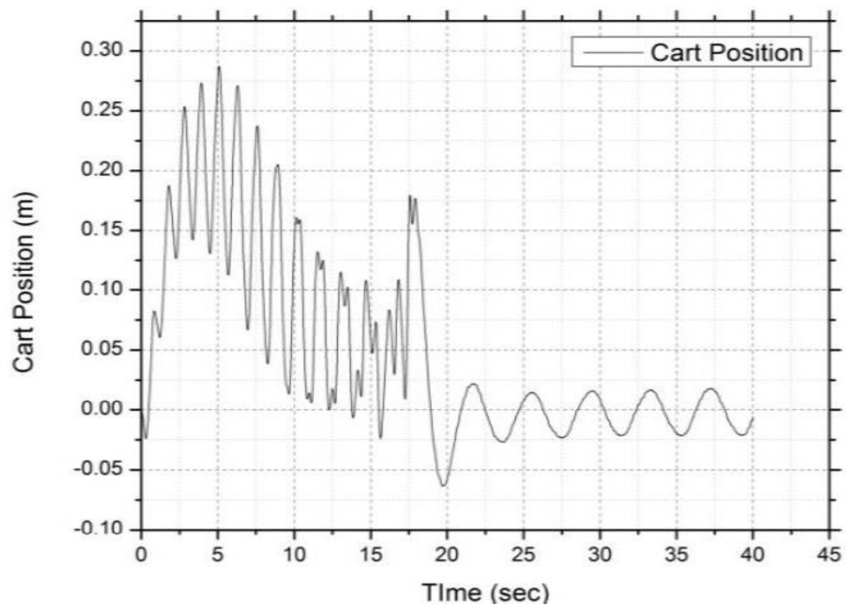


Figure 3-10 Cart position for PID controller

For PID (Figure 3-10) controller we observe that cart moves rapidly with high amplitude during transients when it is trying to swing up the pendulum at an inverted position from its equilibrium position. Once desired position is reached, cart movement stabilizes and now has much lesser amplitude.

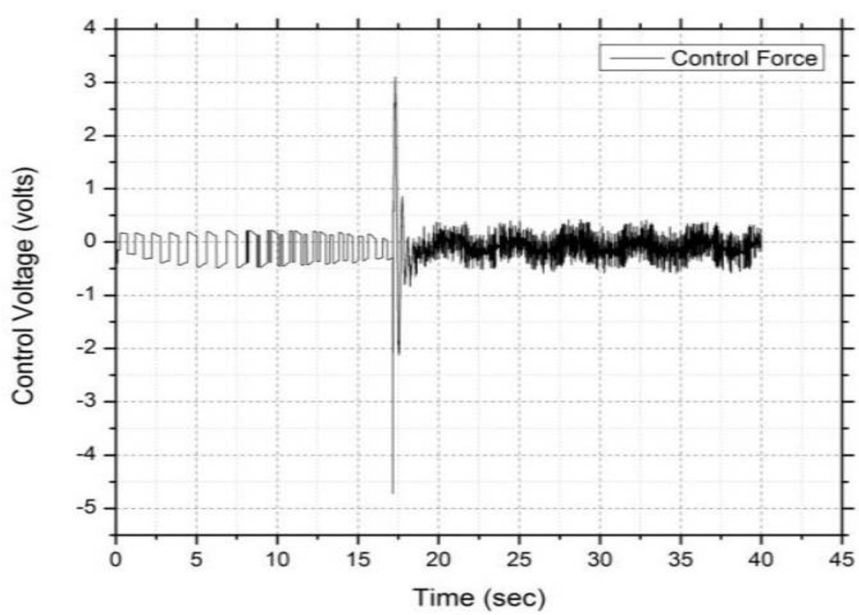


Figure 3-11 Control force for PID controller

Dynamics of controller output are opposite when compared to "cart movement" or "angular position" (Figure 3-11). When cart is moved the controller force varies gradually to move the cart to one end, then controller waits for the pendulum gain inertia of and cart is moved momentarily to the opposite side. When the pendulum reaches desired position the cart needs an impulsive push in order to counteract the pendulum's inertia, which is evitable from the controller response around settling time. Once the desired pendulum angle is reached the controller job becomes more challenging as the pendulum needs to be balanced at an inverted position (opposite to its natural equilibrium) while the gravity constantly pulls the pendulum down. Due to this the controller output changes rapidly. Table 3-3 and Table 3-4 depict the controller performance parameters:

Table 3-3 Settling time and peak value of pendulum angle for PID controller

Controller	t_s(Sec)	M_p(Radians)
PID	17.3	6.01

Table 3-4 Performance indices for PID controller

Error indices	ISE	ITSE	IAE	ITAE
PID	204.3	1902	54.75	477

3. 4. 2 Real-time fuzzy logic control

The fuzzy logic control architecture is given in Figure 3-2. This control architecture is employed for realtime pendulum angle control for a DPCS system and response is illustrated in Figure 3-12.

Figure 3-12 shows the response of pendulum angle for fuzzy logic control with default parameters. The natural equilibrium is considered as the pendulum initial position (i.e. angle =180°). The pendulum is brought to an inverted position with help of back and forth cart movements. The desired inverted position is achieved quickly as compared with PID controller and the settling time is 9.9 seconds less than PID controller.

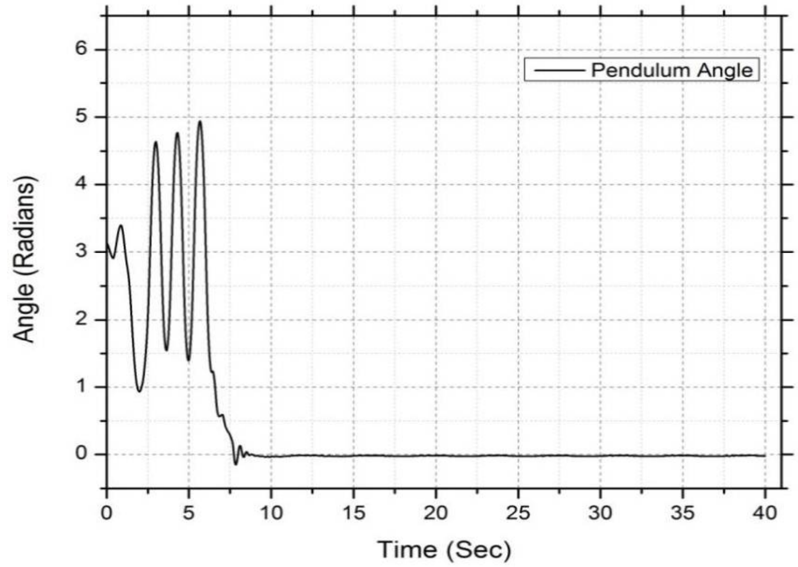


Figure 3-12 Pendulum angle for fuzzy logic controller

The rapid responses for FLC are reflected in the “cart position” & “control voltage” responses which are illustrated in Figure 3-13 and Figure 3-14 respectively.

In Figure 3-13 we observe that the cart movements are rapid and have high amplitude during transients. Once the pendulum reached an inverted position, the cart movement also stabilizes and now has a much lesser amplitude as the controller tries to sustain the desired inverted position.

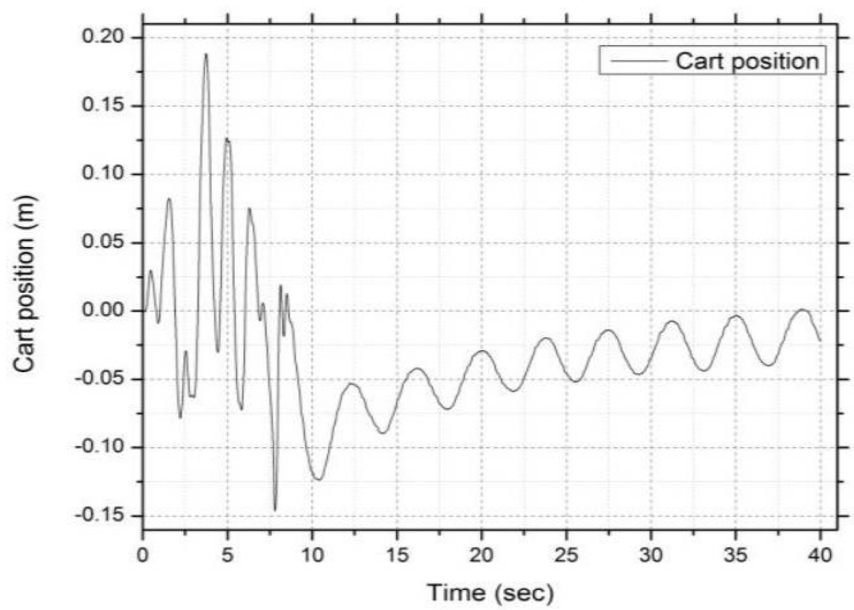


Figure 3-13 Cart position for fuzzy logic controller

A similar change is noticed in control voltage, where dynamics of the controller output are opposite when compared with cart movement dynamics or the pendulum angular position (Figure 3-14).

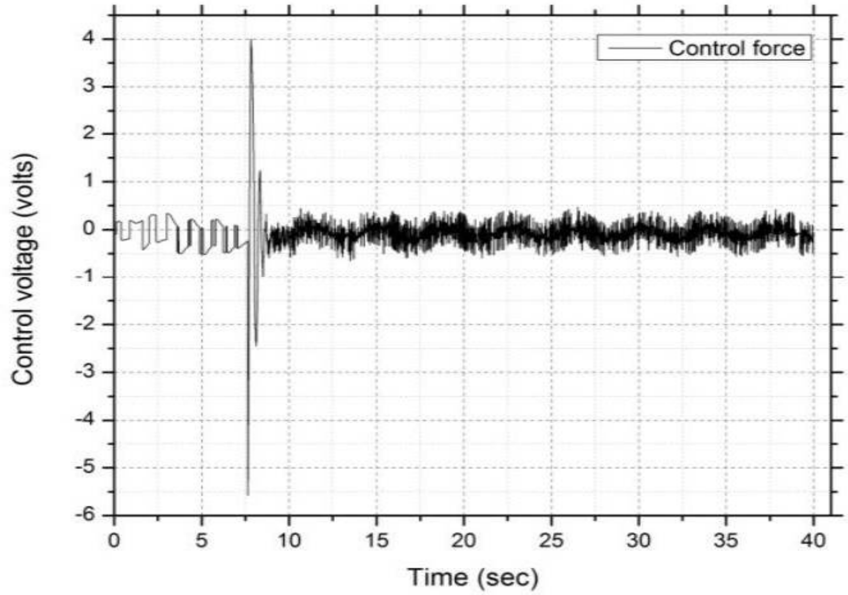


Figure 3-14 Control force for fuzzy logic controller

Here too we observe that the cart is moved gradually in order to swig up the pendulum to its desired position. When the pendulum hits the desired position the cart experiences an impulsive push so as to counteract the pendulum's inertia, which is evident from response pattern post settling time. Once the desired pendulum angle is reached the controller job becomes more challenging since the pendulum has to be "balanced" at desired position (opposite to its natural equilibrium) while the gravity constantly tries to pull down the pendulum, therefore the controller output changes rapidly. Table 3-5 and Table 3-6 depict the controller performance parameters:

Table 3-5 Settling time and peak value of pendulum angle for fuzzy logic control

Controller	t_s(Sec)	M_p(Radians)
FLC	7.4	4.9

Table 3-6 Performance indices for fuzzy logic control

Error indices	ISE	ITSE	IAE	ITAE
FLC	60.99	215.2	19.72	83.95

3. 4. 3 Real-time optimized fuzzy logic control

The optimized values for FSs are calculated using the proposed optimization algorithm and the initial sets are changed to optimized sets thus obtained. This optimized controller is now utilized for real time pendulum angle control for DPCS system and response obtained is depicted in Figure 3-15.

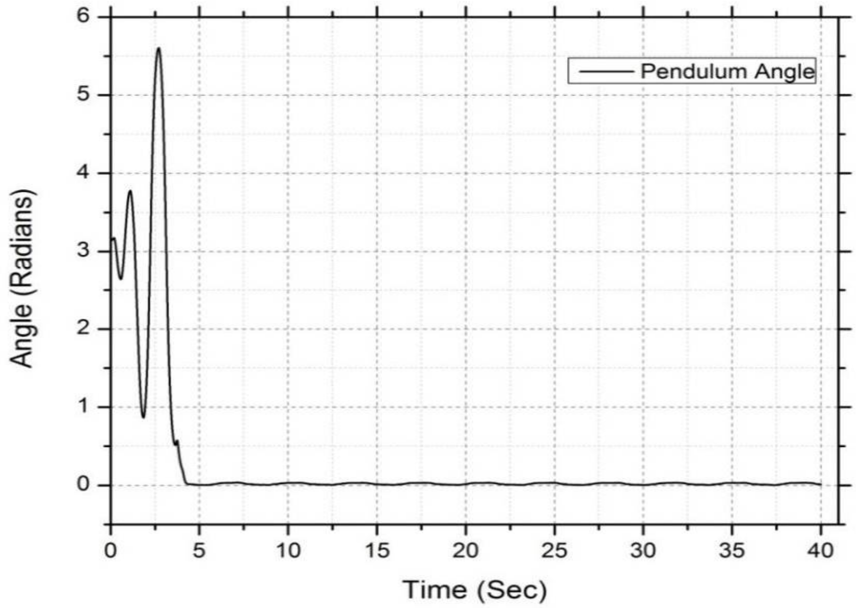


Figure 3-15 Pendulum angle for optimized fuzzy logic controller

The real-time experiment with identical initial condition is repeated, here the pendulum is considered to rest at its natural equilibrium position i.e. angle =180°. The pendulum is brought to inverted position with help of back and forth cart movements. The desired inverted position is achieved quickly as compared with PID controller and the settling time is 12.7 seconds less than PID controller and 2.8 seconds less when compared to FLC. The fast response of FLC is also reflected by “cart movement” & “control voltage” as depicted in Figure 3-16 and Figure 3-17 respectively.

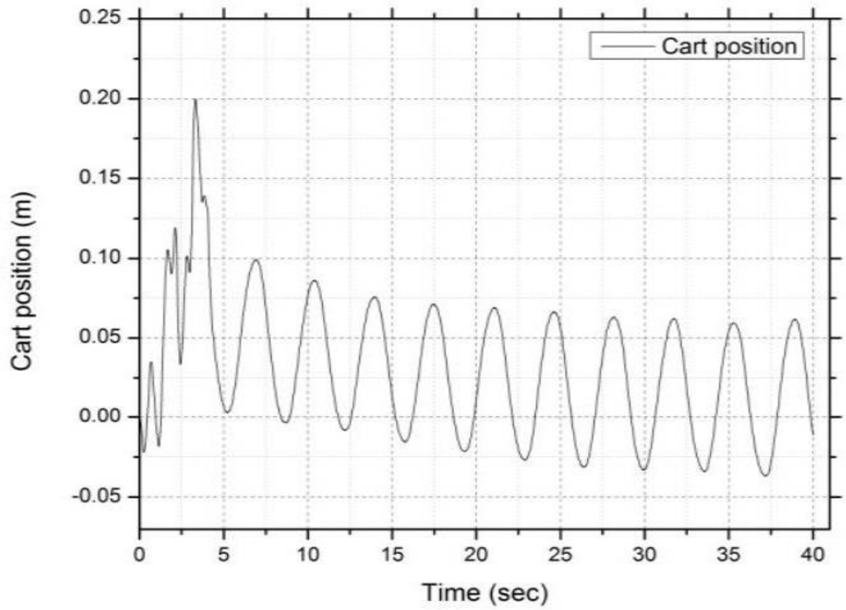


Figure 3-16 Cart position for optimized fuzzy logic controller

In Figure 3-16 we observe that the cart moves rapidly with high amplitude during the transients. Once inverted angular position is achieved, cart movement also stabilizes and now has much lesser amplitude as the cart stabs to maintain the inverted position.

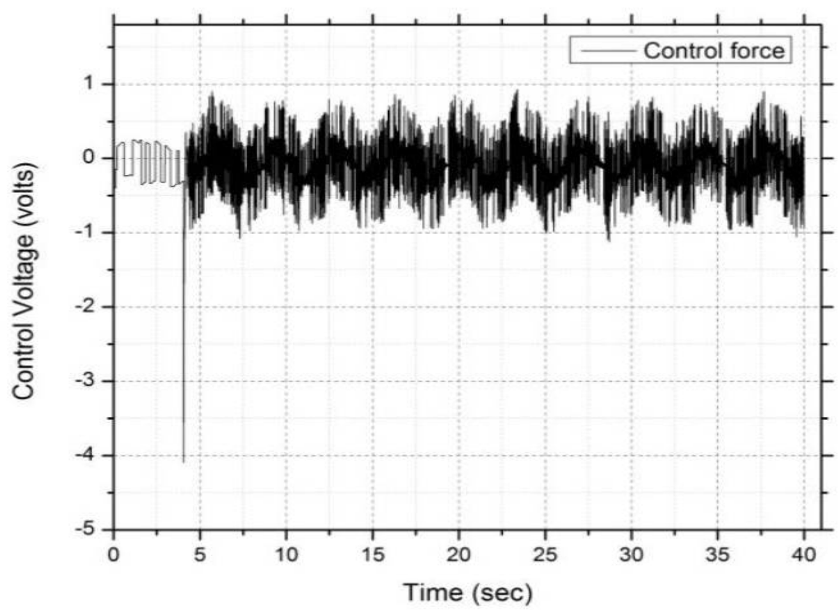


Figure 3-17 Control force for optimized fuzzy logic controller

Here too we observe that the cart is moved gradually in order to swig up the pendulum to its desired position. When the pendulum hits the desired position the cart experiences an impulsive push to counteract pendulum's inertia, and is evident from control voltage response post the settling time. Once the desired pendulum angle is reached the controller job becomes more challenging since pendulum is to be balanced opposite to its natural equilibrium while the gravity constantly pulls the pendulum down. Due to this the controller output changes rapidly. Table 3-7 and Table 3-8 depict the controller performance parameters:

Table 3-7 Settling time and peak value of pendulum angle for optimized fuzzy logic control

Controller	t_s (Sec)	M_p (Radians)
Optimized FLC	4.6	5.52

Table 3-8 Performance indices for optimized fuzzy logic control

Error indices	ISE	ITSE	IAE	ITAE
Optimized FLC	39	76.17	11.57	34.99

3. 5 Comparative analysis

Figure 3-18 depicts one-to-one comparison for the responses obtained from PID controller, FLC and optimized FLC. This figure depicts the responses obtained for real-time swing up stabilization for inverted pendulum with initial condition of pendulum angle =180°.

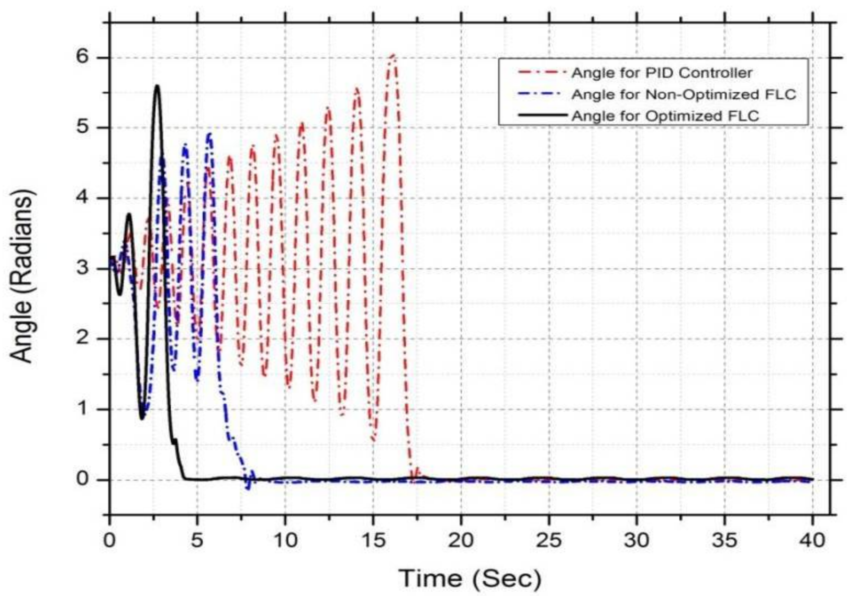


Figure 3-18 Pendulum angle response comparison for PID, FLC and optimized FLC

Real time experiments indicate an "improvement" in the transient response for optimized fuzzy logic control as compared with PID control or fuzzy logic control. The settling time for proposed optimized FLC is 12.7 seconds faster when compared with PID controller and is 2.8 faster as compared with FLC.

The peak overshoot exhibited by optimized FLC is 0.49 radians less when compared to PID controller, however it is 0.62 radians higher when compared with FLC. The cart position response and generated control force PID, FLC and optimized FLC are given in Figure 3-19 and Figure 3-20 respectively.

To bring the pendulum to desired position the cart moves back and forth (illustrated in Figure 3-19) and swings the pendulum and uses its inertia to attain the inverted position. The dynamics of pendulum angle response is reflected by "cart movement" also. The transients occurring in the angular position response produce rapid cart movements which are slows down once the steady-state is reached (i.e. desired position).

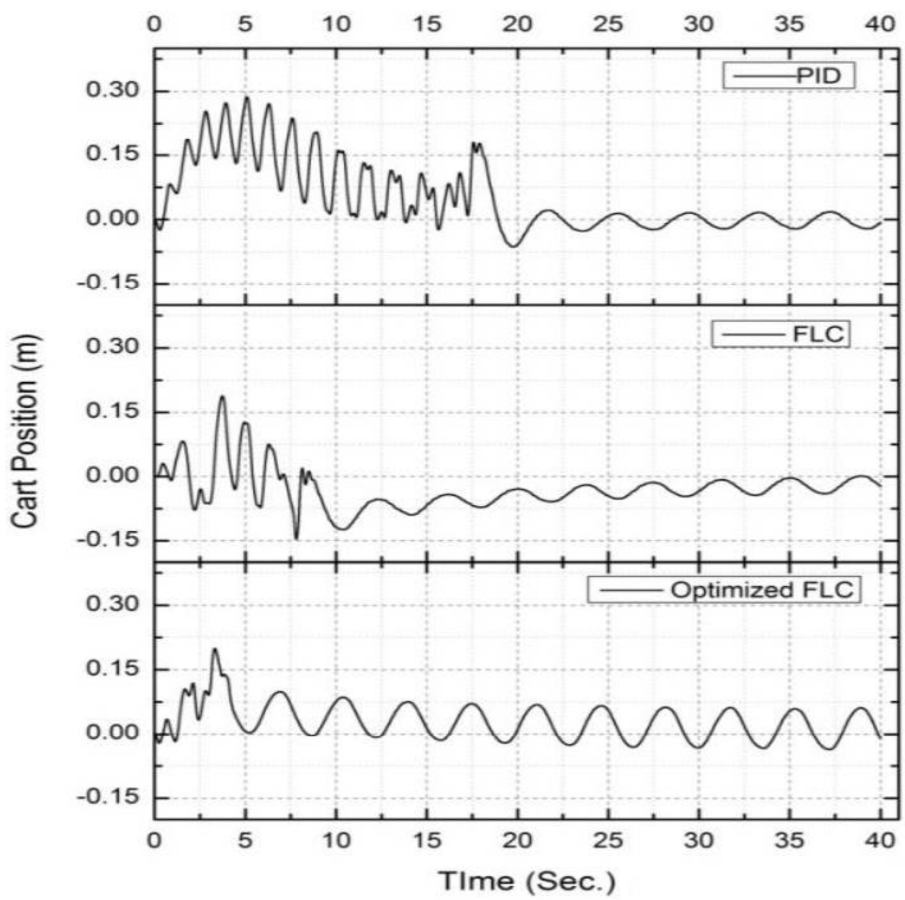


Figure 3-19 cart position comparison for PID, FLC and optimized FLC

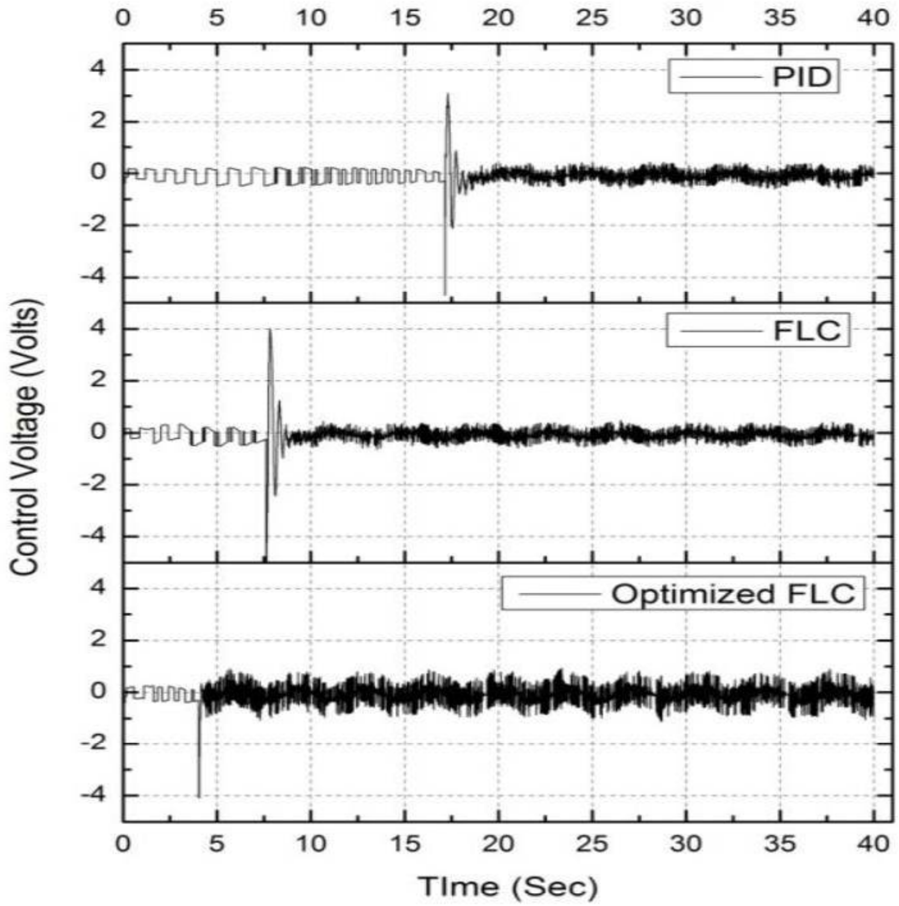


Figure 3-20 control force comparison for PID, FLC and optimized FLC

Performance parameters for PID control, fuzzy logic control, and proposed optimized fuzzy logic control are assessed in Table 3-9and Table 3-10. These parameters evaluated form the experimental response validates that the performance for optimized fuzzy logic control is superior as compared with PID control, or fuzzy logic control (non-optimized).

Table 3-9 Settling time and peak value comparison

Controller	t_s (Sec)	M_p (Radians)
PID	17.3	6.01
FLC	7.4	4.9
Optimized FLC	4.6	5.52

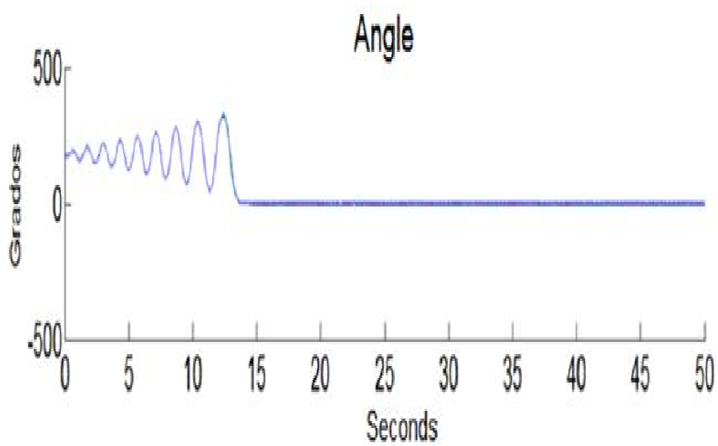
Table 3-10 Performance indices comparison

Error indices`	ISE	ITSE	IAE	ITAE
PID	204.3	1902	54.75	477
FLC	60.99	215.2	19.72	83.95
Optimized FLC	39	76.17	11.57	34.99

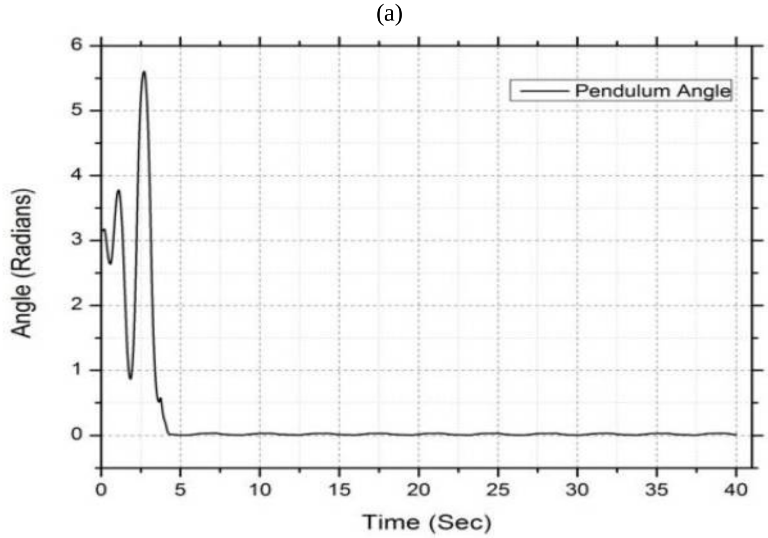
The main advantage of the proposed algorithm is a significant reduction in rise time and turning system to be more responsive. The error indices are also reduced for optimized FLC.

3. 5. 1 Comparison of proposed controller with referenced work

Performance for the proposed controller is also compared with some referenced work employing FLC for swing up control of inverted pendulum system. Complexity of a fuzzy system increases with an increase in the input variables. The controller proposed by Ochoa [94] consists of four inputs as compared with two inputs FLC proposed in this research, thereby reducing complexity. However the fuzzy sets used in Ochoa's controller were 2 for each input and in proposed work are 7 for each of the input, hence resulting in the 49 rules for proposed controller as compared with 16 rules for the former.



(a)



(b)

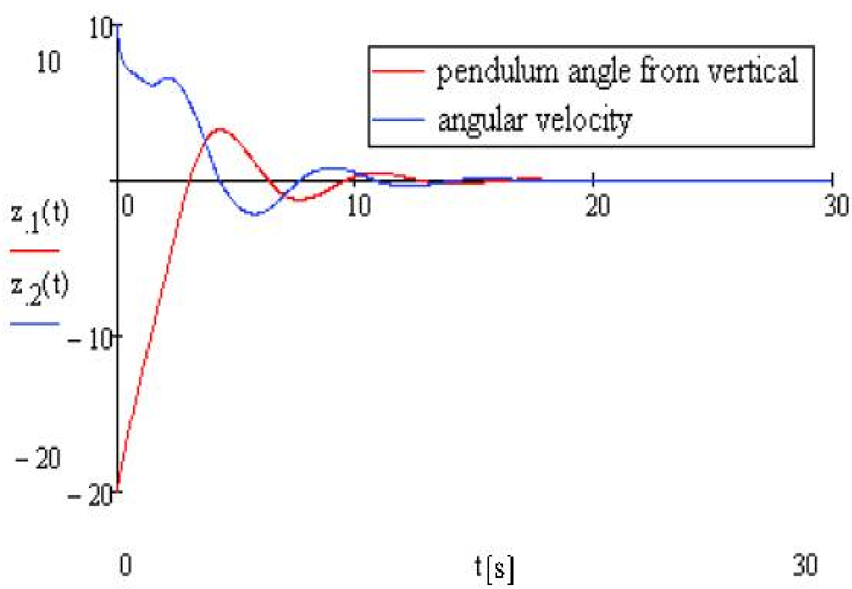
Figure 3-21 Swing up stabilization response (a) Precup's controller [94] (b) proposed controller response

Figure 3-21 depicts the comparison for response of proposed controller with Mamdani FLC developed by Ochoa utilizing Gaussian MFs. The comparison clearly indicates better response for the proposed controller. Proposed controller exhibits 64.06% faster settling time when compared to Ochoa's controller. The settling time for both techniques is illustrated in Table 3-11.

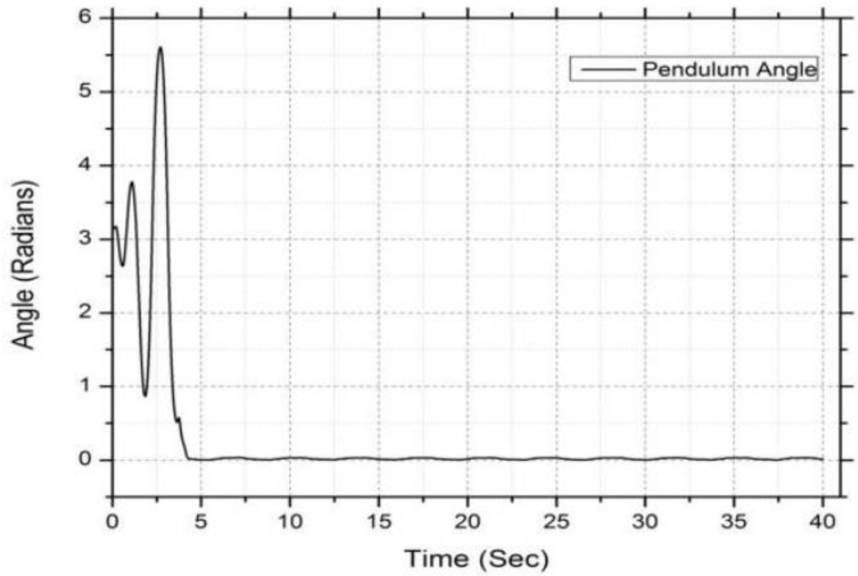
Table 3-11 settling time comparison of proposed controller with reference controller

Controller	t_s(Sec)
Proposed	4.6
Gaussian MF based FLC [94]	12.8

Precup et.al. [95] propsoed a Takagi-Sugeno based FLC for swing up control for inverted pendulum. Precup analyzed the stability of proposed controller using Lyapunov's direct method. The number of input variables is same for Precup's controller and proposed controller, utilizing "error" & "rate of change of error" as inputs. However the fuzzy sets used in Precup's controller were 3 for each input and in proposed work are 7 for each input, resulting in 49 rules for proposed controller as compared with 9 rules for Precup's controller. Response comparison is illustrated in Figure 3-22.



(a)



(b)

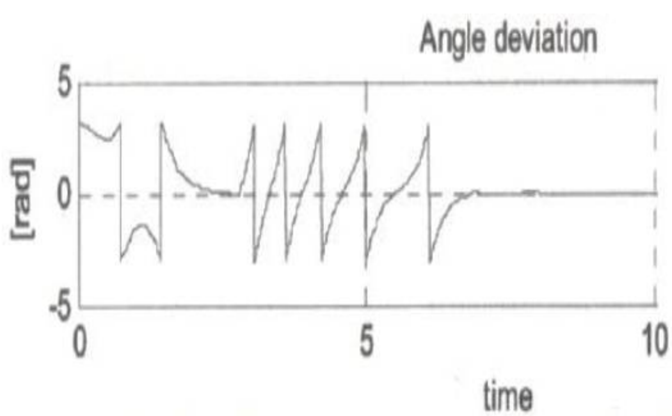
Figure 3-22 Swing up stabilization comparison of (a) Takagi-Sugeno FLC based on Lypaunov's direct method **[95]** (b) proposed controller response

Settling time for proposed controller and Precup's controller response are depicted below in Table 3-12. The settling time achieved for proposed controller is 47.13% faster as compared with the Precup's controller.

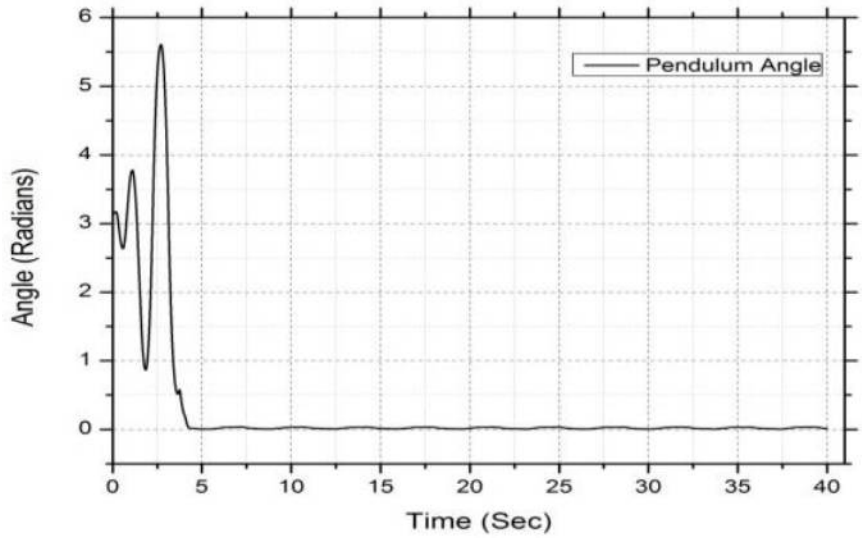
Table 3-12 settling time comparison of proposed controller with reference controller **[95]**

Controller	t_s(Sec)
Proposed	4.6
Lypaunov's based FLC [95]	8.7

Alexander K. Ichtev [96] proposed a Takagi-Sugeno based FLC for swing up control of inverted pendulum. Alexander's FLC has two inputs:"error in pendulum angle" & "rate of change of error", having 3 triangular MFs each, and leading to 9 rules. Figure 3-23 depicts the swing up stabilization response comparison to the proposed controller.



(a)



(b)

Figure 3-23 Swing up stabilization comparison of (a) proposed controller response with (b) Takagi-sugeno FLC response **[96]**

Here it can be observed that the proposed controller exhibited a 31.58% faster settling time when compared to Ichtev's controller. The settling time comparison for proposed controller and Ichtev's controller is depicted in Table 3-13.

Table 3-13 settling time comparison of proposed controller with reference controller **[96]**

Controller	t_s(Sec)
Proposed	4.6
PD type FLC, triangular MF	7

3. 6 Conclusion

The result analysis and comparison for swing up stabilization of inverted pendulum is evaluated by comparing the "settling time" exhibited in achieving the desired position. The controllers discussed in above section exhibit "0" steady-state error.

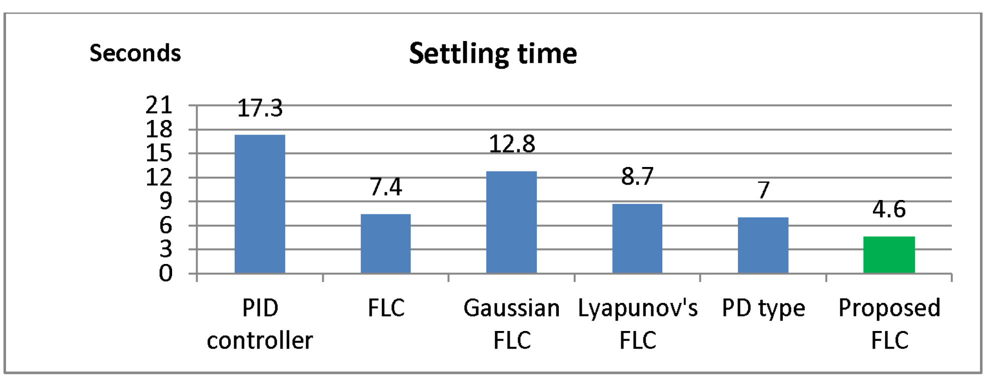


Figure 3-24 Settling time comparison of proposed controller with reference controllers

Figure 3-24 depicts the comparison of settling time for proposed controller with PID controller, FLC (non-optimized) and reference controllers illustrated above. The proposed controller exhibited a faster settling time which implies faster response time and reduction in “control effort”. The reduction observed in error indices clearly indicates faster achievement of inverted position, which is due to lesser overshoot and settling time. Table 3-9 and Table 3-10 summarize the settling time, peak overshoot values and error indices respectively. Proposed controller exhibits a reduction of 73.41% in “settling time” as compared with PID and a reduction of 37.83% compared to FLC. Similarly the peak overshoot is now less by 8.15% as compared with PID but is 11.23% more as compared to FLC. The comparison in “settling time” also shows the faster response for optimized FLC. The reduction in error indices clearly indicate the improvement in response for optimized FLC. Proposed controller exhibits a reduction of about 81.02% in ISE as compared with PID and is less by 36.05% when compared to FLC. The ITSE is less by 95.99% as compared with PID and 64.60% as compared with FLC. IAE shows a reduction of 78.86% as compared with PID and a reduction of 41.32% as compared with FLC. Similarly ITAE shows a reduction of 92.66% as compared with PID and a reduction of 58.32% when compared to FLC. The reduced error indices indicate the steady-state is achieved faster for proposed controller as compared with PID or FLC. It also indicates a zero steady-state error as the time penalized indices also displays a drastic reduction.