

Chapter 2

Fourier Based OFDMA

2.1 INTRODUCTION

Frequency-division multiplexing (FDM) takes a parallel data stream and divides it into N frequency sub channels which do not spectrally overlap and separated by a guard band. The separation of the sub channels is possible using three available schemes [29]:

1. Using filters to completely separate sub-channels. This method was taken from the conventional FDM technology. The limitation of filter implementation is that it forces the bandwidth of each sub-channel to be equal to $(N + \beta) \cdot f_n$, where N is the number of subcarrier's, β stands for the roll-of-factor and f_n is the Nyquist bandwidth. And also, when the number of sub-carriers are large it is difficult to assemble the set of matched filters which is the another disadvantage of using filters.
2. Using staggered Quadrature Amplitude Modulation (QAM) to increase the efficiency of band usage. The modulated sub-carriers use an extra bandwidth of ΔB and are overlapped at the 3 dB frequency. The advantage is that the composite spectrum will be flat.
3. Using the Discrete Fourier transform (DFT) for the modulation and demodulation of the parallel data. The individual spectra are not band limited since they are sinc functions now. The FDM is achieved by band pass processing, not by bandpass filtering. The main advantage of this method is that the transmitter and the receiver can be implemented by using FFT algorithm which reduces the number of complex operations from N^2 to $N \log_2 N$.

In the FDM, guard spacing between the sub-channels is necessary in order to avoid spectral overlap of channels to overcome the problem inter-channel interference. The available spectrum is not efficiently being used in FDM technique, to cope with this problem, an sub-channel overlapping multicarrier

modulation technique, can be used and that is OFDM. By using this technique, almost half of the bandwidth can be saved which can be used for transmitting other sub-channels [30]. This is also illustrated in Fig. 1.1 and 1.2. The Multiplexing concept of OFDM is achieved by IFFT and FFT operations at the transmitter and receiver respectively. The advantage of using FFT over DFT is the fact that DFT uses N^2 complex multiplications where as FFT uses $N\log_2N$ [31], [32]. Multiple access schemes, such as Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), Coded Division Multiple Access (CDMA), and Space Division Multiple Access (SDMA) can be implemented in OFDM systems in the same way as in a single carrier system [33], [34], [35]. Among them, the blend of FDMA and OFDM results in OFDMA, which has been widely considered as one of the most favorable multiple access schemes for next-generation systems [36].

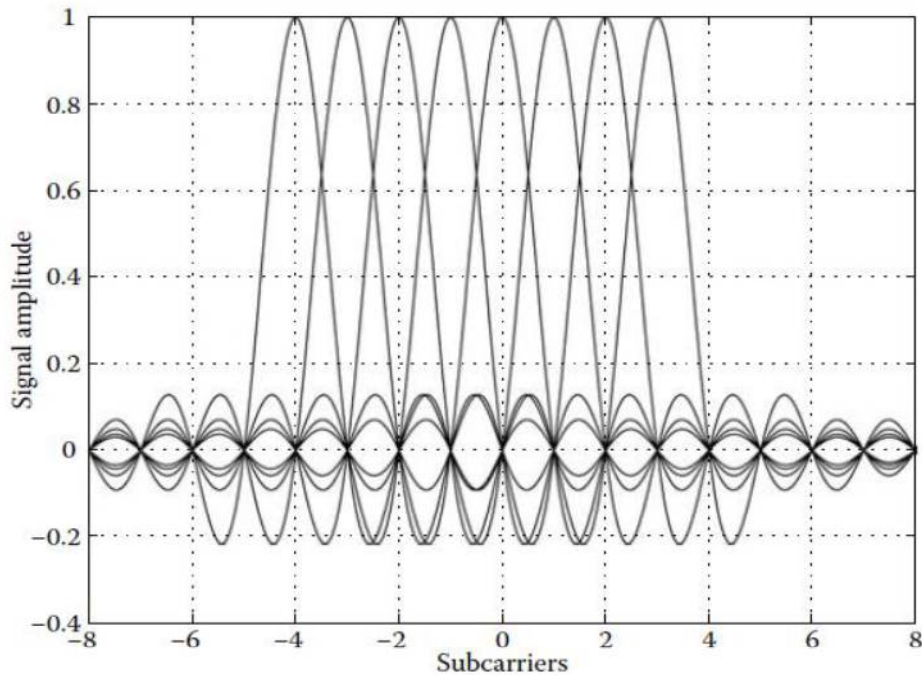


Fig.2.1. OFDM spectrum with 8 Sub Carriers

On the other hand, OFDMA is an extension of the OFDM onto a multiuser environment. The alluring advantages of OFDMA are, including its scalability, intrinsic protection against Multiple Access Interference (MAI), flexible management of the resource. OFDM/A is more robust to multipath channels and simplified equalization has adopted in a wide range of systems, such as the IEEE 802.20 [37], which will be the true mobile wireless technology with high capacity and high mobility like European Telecommunication Standards Institute (ETSI) broadband radio access networks [38], multiuser satellite communications [39], and the fourth-generation cellular network [40]. The OFDMA subcarrier structure can support a wide range of bandwidths, by adjusting FFT size to channel bandwidth while fixing the subcarrier frequency spacing. In this way, the basic unit of physical resource is fixed and the impact to higher layers is minimized. This scalability is one of the most important advantages offered by OFDMA and allows flexibility in deployment [41].

In practical implementation, parallel lower rate data streams are formed from the incoming data sequence, which are modulated onto one of the subcarriers by IFFT operation. Inter-Symbol Interference (ISI) and Inter-Carrier interference (ICI) can be eliminated by inserting cyclic prefix (CP) which creates a cyclically extended guard interval. So that both the ISI and the ICI can be completely eliminated as long as the CP is longer than the maximum delay spread of the channel [42].

This Chapter is organized as follows. The next section basically describes the system process of FFT based OFDM system followed by the section 2.3 in which sub-carrier allocation of OFDMA is discussed. Section 2.4 discusses how orthogonality between sub carriers is achieved and Section 2.5 briefs about the FFT and IFFT i.e. transceiver of OFDM/A. Section 2.6 analyses about the use of cyclic prefix and the last section gives the introduction to 2D-FFT algorithm followed by the summary of this chapter.

2.2 SYSTEM PROCESS OF FFT-OFDMA

An OFDM transceiver is shown in the Fig.2.2.and the Fig.2.3.shows OFDMA transceiver. Sequence of binary numbers is produced from a signal generator, as shown in the Fig.2.2. Input data 'd' consists of binary 0s and 1s. Depending on the system transmission rate, the output from the signal generator will determine the bit rate for the transmission of the signal. As an example, if the system requires 64-QAM then the binary output will have 6 bits per OFDM symbol. The binary number is in multiple of 2 since it has a random probability of a 0 or 1. From this example, it has been observed that there are 6 bits for each OFDM symbol. Using randint built-in function if MATLAB produces a random integer, the integer will be having a random probability of a number between 0 and 63. The mapping process is done after generating binary number d . An OFDM symbol is transformed from a binary to a complex signal with an amplitude and phase represented by a real and imaginary number. This is then encoded and interleaved so as to produce a matrix to be mapped onto QAM values in the form of amplitude and phase. In the case that the system uses 64-QAM, it maps 6 binary bits to 64 constellation points, X_m . This involves taking N parallel streams of QAM symbols (N being the number of sub-carriers used in the transmission of the data) and performing an IFFT operation.

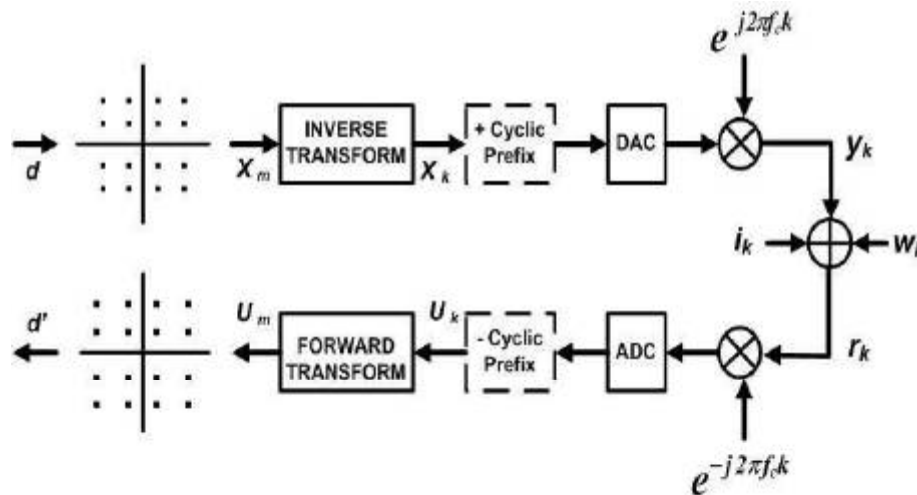


Fig.2.2. OFDM Transceiver illustrating process of OFDM in AWGN channel where W_k represents the Gaussian noise.

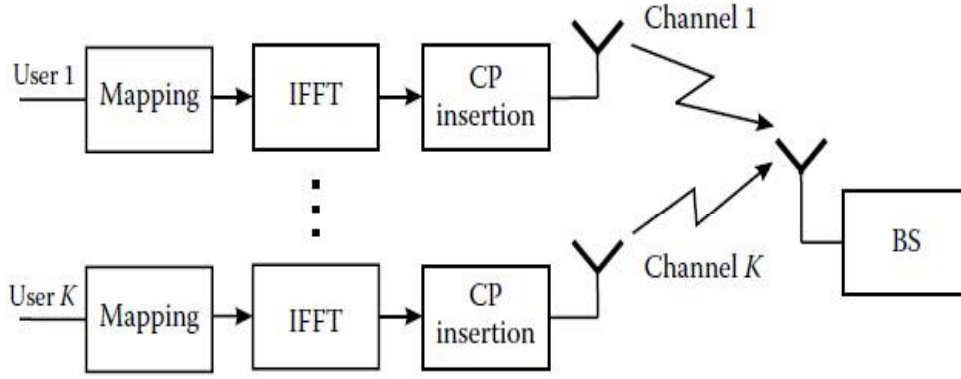


Fig.2.3. Block diagram OFDMA uplink transmission

The output of the IFFT operation in discrete time is as follows:

$$X_k(n) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_m(i) e^{j2\pi \frac{n}{N} i} \quad (2.1)$$

Where $X_m(i)$, $0 \leq i \leq N-1$ represents complex numbers in the discrete frequency domain and $X_k(n)$, $0 \leq n \leq N-1$ represents a sequence in the discrete time domain.

And lastly cyclic prefix (CP) is added before the transmission of the data in order to minimize the inter symbol interference (ISI). At the receiver the process is in the reverse order of the transmission side. The cyclic prefix will be removed to obtain the data in the discrete time domain and then given to the FFT processing to retrieve the original data. The output of the FFT in the frequency domain is given as

$$U_m(i) = \sum_{n=0}^{N-1} U_k(n) e^{-j2\pi \frac{n}{N} i} \quad (2.2)$$

2.3 SUB CARRIER ALLOCATION OF FFT-OFDMA

The objective of OFDMA is to arrange the available subcarriers into several mutually exclusive groups (i.e., sub-bands) according to the strategies of subcarrier allocation. Then each group of subcarriers is assigned to one user for

simultaneous transmission. The orthogonality in the subcarriers takes care that users are protected from Multiple Access Interference (MAI).

Three commonly used carrier allocation schemes (CAS) are illustrated in Fig. 2.4. In the sub-band CAS (SCAS) systems, all the subcarriers of each user are formed as a group and thus by using filter banks the signals can be easily separated. However, the disadvantage is that this scheme does not exploit frequency diversity and a deep fading effect can affect a substantial number of subcarriers for a given user. The interleaved CAS (ICAS) overcomes this problem by allocating subcarriers with uniform spacing's. However, it still has restriction on the resource allocation. The most desirable and flexible method is the generalized CAS, in which users are allowed to select the best available subcarriers to transmit their information, and thereby fully exploits channel frequency diversity and provides also high flexibility in resource allocation [43].

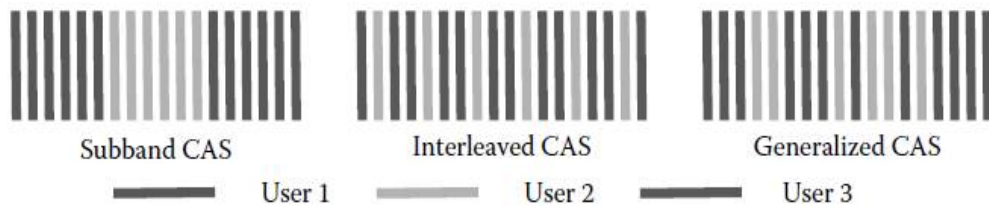


Fig.2.4. Three subcarrier allocation schemes: sub-band CAS; interleaved CAS; generalized CAS.

2.3.1 Transmitter:

At the transmitter of the OFDMA system, the overall N subcarriers are allocated to K users depending on the specific CAS. After that the data sequence of each user will be divided into blocks of length M . The M data symbols in the each block are modulated onto their corresponding subcarriers.

For downlink transmission, other subcarriers in the length N block are modulated with data from other users. The resultant length N block is sent to the conventional OFDM modulator, that is the N -point IFFT. A single CP is added at the front of the OFDM symbol before it is transmitted over K channels to K different users. Downlink transmitter is demonstrated in Fig. 2.5. Unlike the

downlink transmitter, each uplink transmitter left the other subcarriers empty. The resultant length N block with M nonzero entries is sent to an N -point IFFT, followed by the insertion of a distinct CP before it is transmitted through a distinct channel to the base station (BS). The diagram of the OFDMA uplink transmitter is shown in Fig. 2.3.

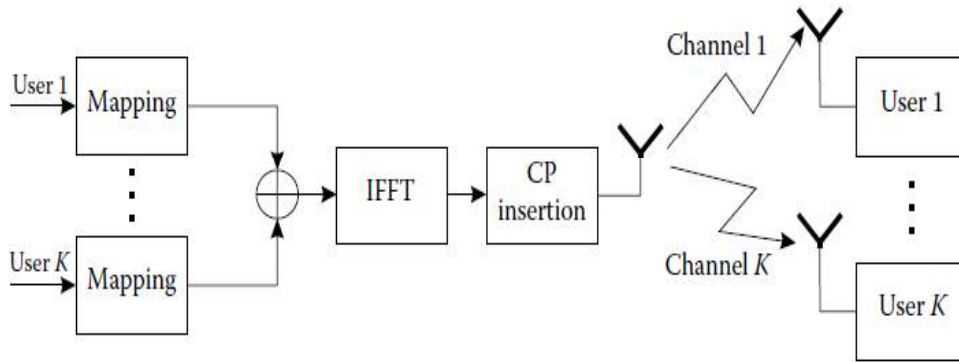


Fig.2.5. Block diagram of OFDMA downlink transmission.

2.3.2 Receiver:

At the receiver, firstly the cyclic prefix is removed, and the remaining N samples are passed to an N -point FFT. One of the supreme advantages of OFDMA is that it effectively converts a wide frequency selective fading channel into a set of parallel flat-fading channels. Thus, by using a bank of one-tap multipliers channel equalization can be easily performed, one for each subcarrier. The output of the equalizer is then sent for detection of data. For downlink transmission, each user only picks up the M data symbols transmitted over its allocated subcarriers for channel equalization and data detection. For uplink, the received signal at the BS is the superposition of signals from all active users, which pass through different channels and experience different frequency offsets and propagation delays. Signals from different users can only be separated after correcting the offsets of individual users. Thus, all the parameters are estimated jointly and this makes the synchronization of OFDMA uplink a very difficult problem.

2.4 ORTHOGONALITY

Orthogonality for an OFDM signal is a concept of multiple overlapping information signals that are transmitted perfectly without suffering to the interference. This can be done when the signals have integer number of cycles per OFDM symbol. By having them cyclic, it allows an integer number of samples within the IFFT/FFT interval. The mathematical relationship between the subcarriers to safeguard orthogonality in the system is given in the equation 2.3.

$$\int_0^{T_s} \sin(n\omega t) \sin(m\omega t) dt = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases} \quad (2.3)$$

Where $\omega = 2\pi f$, f is the carrier spacing, m and n are the number of carriers and T_s is the OFDM symbol period excluding the guard interval. The equation (2.3) is the same principle as the equation defined in [44] which states that the integral indicates that the integer number n and m must not be equal so that the integration is zero. Then only the orthogonality property is satisfied with integration over a period of symbol duration T_s . If the integer n is equal to m , the integration over a period T_s cannot be zero, thus, it will not be orthogonal.

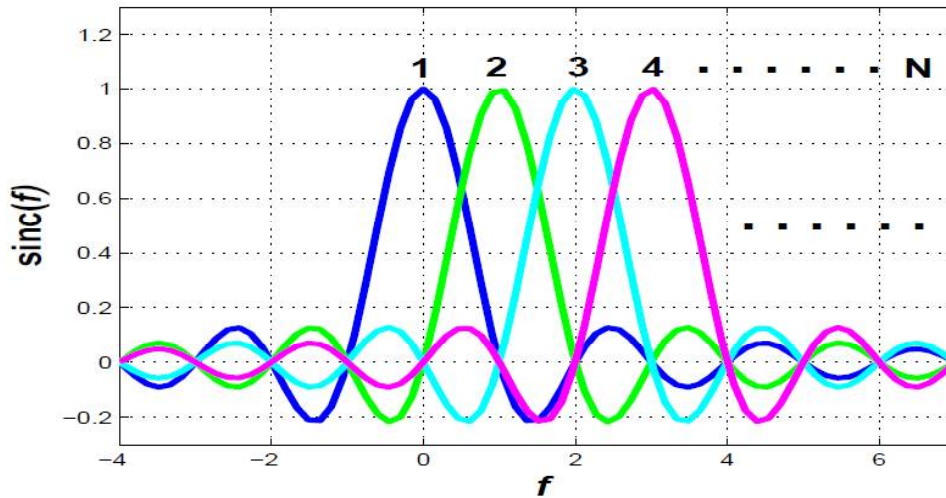


Fig.2.6. Overlapping spectra of subcarriers per OFDM symbol.

The orthogonality property of OFDM signals can also be viewed by looking at its frequency spectrum. Fig. 2.6 shows the first 4 OFDM subcarriers where N represents the number of subcarriers per OFDM symbol. It can be observed that each OFDM subcarrier follows a $\text{sinc}(f)$ or $\sin(x)/x$ spectrum where x is πf . This is the result of the symbol time corresponding to the inverse of the carrier spacing $1/T$ Hz, where T is the duration time for fast Fourier transform which is a fraction of the OFDM symbol period T_s . The spectrum of an effective OFDM signal can be obtained by the convolution of a sinc function, which is the spectrum of a single subcarrier, with a sequence of dirac pulses $\delta(f)$ equi spacing with Δf . The sinc function has a narrow main lobe at the center with many side-lobes that decay slowly with the magnitude of the frequency difference away from the center. Each subcarrier has a peak at the center frequency and nulls evenly spaced with a frequency gap equal to the carrier spacing. From Fig. 2.6, it can be seen that the first subcarrier, peak occurs at 0 subcarrier frequency and zeroes at other subcarrier frequencies 1, 2, 3 until N , the same principle applies to the adjacent subcarriers 2, 3, until N . It shows that the orthogonal nature of the transmission is a result of the peak of each sub carrier corresponding to the nulls of all other sub carriers.

2.5 IFFT AND FFT OPERATIONS

In the Physical layer of OFDMA, performing FFT operation is one of the important tasks. As shown in the Fig. 2.2. IFFT operation will be performed on the parallel data and then cyclic prefix is added to that output samples of the IFFT before transmitting the data into the channel.

The FFT algorithm has advantages when compared to the DFT operation. The advantage is in terms of calculation of complex multiplications. The algorithm states that to perform DFT operation of input samples size N , it requires N^2 complex multiplications whereas IFFT operation uses only $(N/2) \log_2 N$ [45], [46]. The general butterfly structure in the FFT/IFFT is called Radix-2 structure. The

number of calculations will be still reduced as the size of the Radix is increased. It is represented as $(3N/4) \log_4 N$. The comparison is shown in the Table 2.1.

Table 2.1.FFT: Radix-2 and Radix-4 Complex Multiplication Comparisons

N	Radix-2 FFT	Radix-4 FFT
8	12	3
16	32	12
32	80	36
64	192	96
128	448	240
256	1024	576

An example of radix-4 algorithm implementation for N subcarriers in a four point IFFT is shown as follows:

$$\frac{1}{N} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{N} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (2.4)$$

The same equation can be written as

$$\frac{1}{N} A.x = \frac{1}{N}.y \quad (2.5)$$

Where A denotes the matrix consisting of IFFT values, x is the input vector and y is the output vector both are scaled by $\frac{1}{N}$ in which N denotes the number of sub carriers. Now, let us assume that we need to transmit four binary values [-1 1 -1 -1] on the four sub carriers the matrix operation will be given as follows

$$\frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 \\ 2j \\ -2 \\ -2j \end{bmatrix} \quad (2.6)$$

Thus, to form one OFDM symbol there are four IFFT outputs. After performing the IFFT, cyclic prefix is added to the OFDM signal followed by windowing to combat inter-channel interference (ICI). Finally, the calculated time domain signal is mixed along with the required frequency in a RF amplifier component before being passed to the antenna for transmission into the channel. At the receiver side, the reverse operation will take place starting from the removal of cyclic prefix, then FFT operation followed by the demodulation of QAM, deinterleaving and decoding before returning to the original data. However, in order to get more flexibility in terms of computations 2D-FFT algorithm has been used which will be discussed in the section 2.7.

2.6. CYCLIC PREFIX

The OFDM has the capability to deal with interference in an efficient way by using a cyclic prefix/guard time for each of its symbols. In this case, to overcome interference between each OFDM symbols, the guard time must be set at least four times larger than the expected delay spread. A typical OFDM block showing guard time, the effective OFDM symbol period or the FFT/IFFT duration and the total OFDM symbol period is shown in Fig. 2.7.

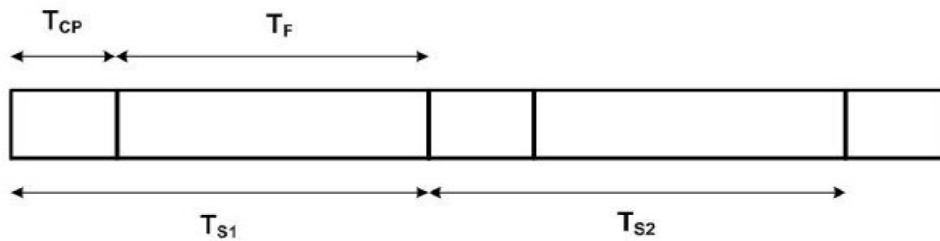


Fig.2.7. Typical schematic of perfect OFDM block showing Guard band T_{cp}

It should be considered that the chosen duration of the guard time must be optimum; otherwise, the OFDM transmission would have lost substantial power. In order to achieve orthogonality the guard time should be assumed to be part of the signal. In the case that there is no signal within the guard time, the orthogonality cannot be achieved in the presence of multipath environments. This

situation results in inter-symbol interference (ISI), inter-carrier interference (ICI) in the form of crosstalk between different subcarriers.

The required spacing in time between the reference symbols can be obtained by considering the maximum Doppler spread (highest speed) to be supported, which for LTE corresponds to 500 km/h. The Doppler shift is $f_d = \frac{f_c v}{C}$, where f_c is the carrier frequency, v is the User Equipment (UE) speed in m/s, and c is the speed of light (3×10^8 m/s). Considering $f_c = 2$ GHz and $v = 500$ km/h, then the Doppler shift is $f_d = 950$ Hz (approx.). According to Nyquist's sampling theorem, the minimum sampling frequency needed in order to reconstruct the channel is therefore given by $T_c = 1/(2f_d)$ which is 0.5ms (approx.) under the above assumptions. This implies that two reference symbols per slot are needed in the time domain in order to estimate the channel correctly.

2.7 2D-FFT ALGORITHM

Long FFTs are quite often used for frequency analysis and communications applications. These long word lengths affect the memory architecture because long word lengths require more memory bandwidth for the matrix transpositions. The solution for such word lengths is implementing the same by using two shorter length FFTs (lengths N_1 and N_2) to calculate an FFT of length $N = N_1 \times N_2$. 2D-FFT algorithm not only improved the system level of the parallelism, but also reduced the demand for memory systems [47].

The two-dimensional (2D) FFT of $N = N_1 \times N_2$ is defined as follows.

$$X[k_1 N_2 + K_2] = \sum_{n_1=0}^{N_1-1} \left[e^{\frac{-j2\pi n_1 k_2}{N}} \left(\sum_{n_2=0}^{N_2-1} X(n_2 N_1 + n_1) e^{\frac{-j2\pi n_2 k_2}{N_2}} \right) \right] e^{\frac{-j2\pi n_1 k_1}{N_1}} \quad (2.7)$$

As an example if 64-pt FFT is to be implemented then it can be implemented by using two shorter FFT's as 8 x 8 point FFT's.

Table.2.2.Comparison of complex multiplications, DFT, Radix-2 FFT & Radix-2 2D-FFT

Number of Points, N	Complex Multiplications in Direct Computation, N²	Complex Multiplications in FFT Algorithm, (N/2) Log₂N	Speed Improvement factor	Complex Multiplications in 2D-FFT algorithm,	Speed Improvement factor over FFT Algorithm
4	16	4	4	2	2
8	64	12	5.3	2	6
16	256	32	8	4	8
32	1024	80	12.8	4	20
64	4096	192	21.3	8	24
128	16384	448	36.6	8	56
256	65536	1024	64	16	64
512	262144	2304	113.8	16	144
1024	1048576	5120	204.8	32	160

Table.2.2. shows the comparison between the computational complexity in complex multiplication calculations of DFT, Radix-2 FFT and Radix-2 2D-FFT algorithms. The speed improvement factor of 2D-FFT shows that how efficient and fast it is. However, the application of 2D-FFT algorithm to implement OFDMA will be discussed in the next chapter.

2.8 SUMMARY

The conventional FDM uses guard spacing in between the carriers to transmit it to the receiver without interference. But lot of bandwidth is wasted in this technique. To cope with problem, orthogonality between the carriers is introduced in the carriers such that the carriers can be overlapped by each other and this technique is called as OFDM. The main processes of physical layer are IFFT at the transmitter and FFT at the receiver. IFFT and FFT are used in the multiplexing of transmitting and receiving OFDM data respectively. Apart from orthogonality, the use of cyclic prefix is also discussed in this chapter. In order to maintain the orthogonality, the cyclic prefix has to be longer than the channel impulse response. And at last introduction and necessity of 2D-FFT algorithm have been discussed which is used to implement the PHY- layer OFDMA.