Name: Enrolment No:	UPES – UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2019

Programme Name: B. Tech. ADE, Mechanical, Mechatronics, ASE, ASE-AVI | **Semester : II**
Course Name : Mathematics II | **Time : 03 hrs**
Course Code : MATH 1015 | **Max. Marks : 100**
Nos. of page(s) : 02

SECTION A

S. No.		Marks	CO
Q 1	Evaluate $\oint_C \vec{F}.d\vec{R}$ by Stoke's theorem where $F=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ and C is the boundary of the triangle with vertices at (0,0,0), (1,0,0) and (1,1,0).	**4**	**CO2**
Q 2	Determine the analytic function $f(z)=u+iv$ if $u=(x-y)(x^2+4xy+y^2)$.	**4**	**CO4**
Q 3	Evaluate using Cauchy's integral formula $\int_C \frac{\sin \pi z+\cos \pi z}{(z-1)(z-2)}dz$ where C is the circle $\lvert z\rvert=4$.	**4**	**CO4**
Q 4	Solve $\frac{d^4y}{dx^4}-m^4y=\cos mx$.	**4**	**CO3**
Q 5	Determine the type of singularity of the function $\frac{1-e^{2z}}{z^4}$.	**4**	**CO4**

SECTION B

Q 6	Find the series expansion of function $f(z)=\frac{z^2-1}{z^2+5z+6}$ about $z=0$ when (i) $\lvert z\rvert<2$ (ii) $2<\lvert z\rvert<3$	**10**	**CO4**
Q 7	Solve: $\frac{d^2y}{dx^2}+4y=\tan 2x$.	**10**	**CO3**
Q 8	Using complex integration method, evaluate $$\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}d\theta .$$	**10**	**CO5**
Q 9	If $F(t)=\frac{e^{at}-\cos bt}{t}$, find the Laplace transform of $F(t)$.	**10**	**CO6**

OR

Q 9	Find Laplace transform of $f(t)=t^2 e^{-2t}\cos t$.	**10**	**CO6**
SECTION C			
Q 10 A	Determine the poles and residues at each pole of the function $f(z)=\frac{z^2-2z}{(z+1)^2(z^2+4)}$ and hence evaluate $\oint_C f(z)\,dz$ where C is the circle $\lvert z\rvert=10$.	**10**	**CO4**
Q 10 B	Using complex integration, prove that $\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^2}=\frac{\pi}{2}$	**10**	**CO5**
Q 11 A	Change the order of integration in $\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dxdy$ and hence evaluate the same.	**10**	**CO1**
OR			
Q 11 A	Evaluate the following by changing into polar coordinates: $\int_0^a\int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dxdy$	**10**	**CO1**
Q 11 B	Find the inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$.	**10**	**CO6**
OR			
Q 11 B	Apply Convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$.	**10**	**CO6**

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SECTION A

S. No.		Marks	CO
Q 1	Apply Gauss divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and S is the surface of a cube bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$.	4	CO2
Q 2	Find an analytic function whose imaginary part is $e^{-x}(x\cos y+y\sin y)$.	4	CO4
Q 3	Evaluate using Cauchy's integral formula $\oint_C \frac{z}{z^2-3z+2}dz$ where C is $\lvert z-2\rvert=\frac{1}{2}$	4	CO4
Q 4	Solve $\frac{d^6y}{dx^6}-\frac{d^4y}{dx^4}=x^2$.	4	CO3
Q 5	Determine the type of singularity of the function $\frac{e^{1/z}}{z^2}$.	4	CO4

SECTION B

Q 6	Find the series expansion of function $f(z)=\frac{7z-2}{z^3-z^2-2z}$ when (i) $0<\lvert z+1\rvert<1$ (ii) $1<\lvert z+1\rvert<3$	10	CO4
Q 7	Solve: $\frac{d^2y}{dx^2}+a^2y=sec\,ax$.	10	CO3
Q 8	Using complex integration method, evaluate $\int_0^{2\pi}\frac{\sin^2\theta}{a+b\cos\theta}d\theta$.	10	CO5
Q 9	If $F(t)=te^{-t}\sin 3t$, find the Laplace transform of $F(t)$.	10	CO6

OR

Q 9	Find Laplace transform of $f(t)=\int_0^t \frac{e^t \sin t}{t}$ dt	**10**	**CO6**
SECTION C			
Q 10 A	Determine the poles and residues at each pole of the function $f(z)=\frac{z-1}{(z+1)^2(z-2)}$ and hence evaluate $\oint_C f(z)dz$ where C is the circle $\lvert z-i\rvert=2$.	**10**	**CO4**
Q 10 B	Apply calculus of residues to prove that $\int_0^\infty \frac{dx}{(a^2+x^2)^2}=\frac{\pi}{4a^3};a>0.$	**10**	**CO5**
Q 11 A	Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ by changing the order of integration.	**10**	**CO1**
OR			
Q 11 A	Find the area lying between the parabola $y=4x-x^2$ and the line $y=x$.	**10**	**CO1**
Q 11 B	Find the inverse Laplace transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$.	**10**	**CO6**
OR			
Q 11 B	Apply Convolution theorem to evaluate $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$.	**10**	**CO6**