Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme Name: B.Tech ASE and B.Tech ASE+AVE **Semester : VI**
Course Name : Applied Numerical Methods **Time : 03 hrs**
Course Code : MATH 307 **Max. Marks : 100**
Nos. of page(s) : 03

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Instructions: Attempt all questions from Section A (Q1-Q5, each carrying 04 marks); Section B (Q6-Q9, each carrying 10 marks); Section C (Q10 & Q11, each carrying 20 marks). Scientific calculators are allowed for the examination.

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q1.	Let $x_0=1.5$ be the initial approximation of a root of the equation $x^2+\log_e x-2=0$. Find an approximate root of the equation using fixed point iteration method (iteration method), correct upto three significant digits.	[4]	CO1
Q2.	Establish the operator relation $E\equiv e^{hD}$, where E and D denote the Shifting and Differential operators respectively. (h is the step-length).	[4]	CO2
Q3.	Show that the matrix $A=\begin{bmatrix}1 & 1 & -1\\ 2 & 2 & 5\\ 3 & 2 & -3\end{bmatrix}$ is not factorable in form of $A=LU$ by Doolittle's method. Find a new matrix B by rearranging the rows of the matrix A so that B is factorable by that method. Give reason for your answer.	[2+2]	CO4
Q4.	Show that the partial differential equation $x^2u_{xx}-2xy\,u_{xy}-3y^2u_{yy}+u_y=0$ is hyperbolic type at every points on xy-plane except for the coordinate axes $x=0$ and $y=0$. Identify the characteristic of the equation on coordinate axes.	[3+1]	CO6
Q5.	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is	[4]	CO5

	$$\frac{dy}{dx}=-ky, \text{ where } k=0.01.$$ Given that $x_0=0$ and $y_0=100$. Determine how much substance will remain at the moment $x=100$, using Modified Euler's method with the step-length $h=100$.		
SECTION B **(Q6-Q8 are compulsory and Q9 has internal choice)**			
Q6.	Derive 2-points Gauss-Legendre formula for $I=\int_{-1}^{1} f(x)dx$, and apply it to evaluate $I=\int_{1}^{2} e^{\frac{-x^2}{2}} dx$.	**[6+4]**	**CO3**
Q7.	Use fourth order Runge-Kutta method to solve for $y(1.2)$, considering step-length $h=0.1$, given that $\frac{dy}{dx}=x^2+y^2$, with initial condition $y(1)=1.5$.	**[10]**	**CO5**
Q8.	Let $x_0=1.6$ be an initial approximation of the root of the following equation. $10\int_{t=0}^{x} e^{-x^2} dt=1$ Use Newton-Raphson method to find a positive root of that equation, correct to six decimal places.	**[10]**	**CO1**
Q9.	Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then apply Gauss-Seidel method to evaluate an approximate solution, taking the initial approximation as $x_1^{(0)}=1, x_2^{(0)}=1, x_3^{(0)}=1$, corrected to three decimal places. $x_1-x_2+5x_3=7$ $6x_1-x_2+x_3=20$ $x_1+4x_2-x_3=6.$ **OR** Show that the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 8 & 22\\ 3 & 22 & 82\end{bmatrix}$ is decomposable by Cholesky method. Hence find the solution of the following system of equations by that method. $x_1+2x_2+3x_3=5$ $2x_1+8x_2+22x_3=6$ $3x_1+22x_2+82x_3=-10.$	**[3+7]**	**CO4**

SECTION-C

(Q10 is compulsory, and Q11.A and Q11.B have internal choices)

Q10. A — [5+5] — CO1

The speeds of an electric train at various times after leaving one station are given in the following table.

Time t (in hour)	0	$\frac{1}{120}$	$\frac{1}{60}$	$\frac{1}{40}$	$\frac{1}{30}$
Speed v (in mph)	0	13	33	39.5	40

Find the distance (in mile), travelled by the train, and acceleration of the train in 2 minutes.

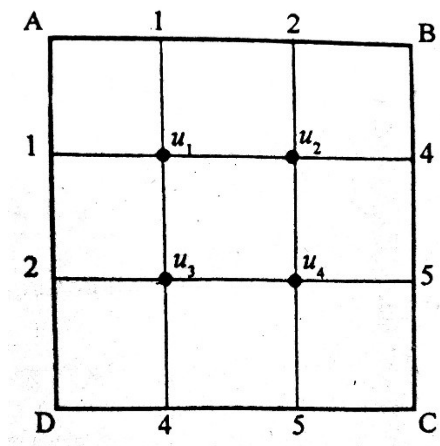
Q10. B — [8+2] — CO2

Fit a polynomial of degree three, which takes the following values, by Newton forward interpolation formula, and find $y(3.5)$.

x	3	4	5	6
y	6	24	60	120

Q11. A — [10] — CO6

Solve the Laplace equation $u_{xx}+u_{yy}=0$ for the following square mesh with boundary values as shown in the figure by Liebmann's iteration process. Perform five iterations.



OR

Solve the Poisson's equation $u_{xx}+u_{yy}=-10(x^2+y^2+10)$ over the square mesh with sides $x=0, y=0, x=3, y=3$ with $u=0$ on the boundary and mesh length 1. Perform three iterations by Gauss Seidal method to solve the linear equations in u.

Q11. B — [10] — CO6

Solve $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t)=0, u(4,t)=0, u(x,0)=x(4-x)$ taking $h=1$ and employing Bender-Schmidt method. Continue the solution through five time steps.

OR

Using Crank-Nicholson's method, solve $u_{xx}=16u_t, 0<x<1, t>0$,

	given that $u(x,0)=0, u(0,t)=0, u(1,t)=50t$. Compute u for two steps in t direction taking $h=\frac{1}{4}$.		

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SECTION A
(Attempt all questions)

S. No.		**Marks**	**CO**
Q1.	Perform four iterations of bisection method to obtain the approximate value of $(17)^{\frac{1}{3}}$ starting with the initial approximation $x_0=2$.	**[4]**	**CO1**
Q2.	Establish the operator relation $D\equiv\frac{1}{h}\ln(1+\Delta)$, where Δ and D denote the forward difference and differential operators respectively. (h is the step-length).	**[4]**	**CO2**

Q3.	Show that the matrix $A=\begin{bmatrix}2 & 2 & 5\\ 1 & 1 & -1\\ 3 & 2 & -3\end{bmatrix}$ is not factorable in form of $A=LU$ by Crout's method. Find a new matrix B by rearranging the rows of the matrix A so that B is factorable by that method. Give reason for your answer.	**[2+2]**	**CO4**
Q4.	Show that the partial differential equation $u_{xx}-y\,u_{yy}+u_y=0$ is hyperbolic type at upper half of xy-plane, elliptic type at lower half of xy-plane and parabolic type on x-axis.	**[3+1]**	**CO6**
Q5.	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is $$\frac{dy}{dx}=-\alpha y, \text{ where } \alpha=0.02.$$ Given that $x_0=0$ and $y_0=100$. Determine how much substance will remain at the moment $x=50$, using Modified Euler's method with the step-length $h=50$.	**[4]**	**CO5**
SECTION B **(Q6-Q8 are compulsory and Q9 has internal choice)**			
Q6.	Derive 2-points Gauss-Legendre formula for $I=\int_{-1}^{1} f(x)dx$, and apply it to evaluate $I=\int_{0}^{1} e^{-x^2}dx$.	**[6+4]**	**CO3**
Q7.	Use fourth order Runge-Kutta method to solve for $y(0.4)$, considering step-length $h=0.2$, given that $\frac{dy}{dx}=1+y^2$, with initial condition $y(0)=0$.	**[10]**	**CO5**
Q8.	Let $x_0=1.6$ be an initial approximation of the root of the following equation. $5\int_{t=2x}^{4x} e^{-x^2}dt=1$ Use Newton-Raphson method to find a positive root of that equation, correct to six decimal places.	**[10]**	**CO1**

Q9.	Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then apply Jacobi method to evaluate an approximate solution, taking the initial approximation as $x_1^{(0)}=1, x_2^{(0)}=1, x_3^{(0)}=1$, corrected to three decimal places. $2x_1+20x_2-2x_3=-44$ $10x_1+2x_2+x_3=9$ $-2x_1+3x_2+10x_3=22.$ **OR** Show that the matrix $A=\begin{bmatrix}4 & 2 & 6\\ 2 & 82 & 39\\ 6 & 39 & 26\end{bmatrix}$ is decomposable by Cholesky method. Hence find the solution of the following system of equations by that method. $4x_1+2x_2+6x_3=16$ $2x_1+82x_2+39x_3=206$ $6x_1+39x_2+26x_3=113$	**[3+7]**	**CO4**

SECTION-C

(Q10 is compulsory, and Q11.A and Q11.B have internal choices)

Q10. A

The speeds of an electric train at various times after leaving one station are given in the following table.

Time t (in hour)	0	$\frac{1}{120}$	$\frac{1}{60}$	$\frac{1}{40}$	$\frac{1}{30}$
Speed v (in mph)	0	13	33	39.5	40

Find the distance (in mile), travelled by the train, and acceleration of the train in 2 minutes.

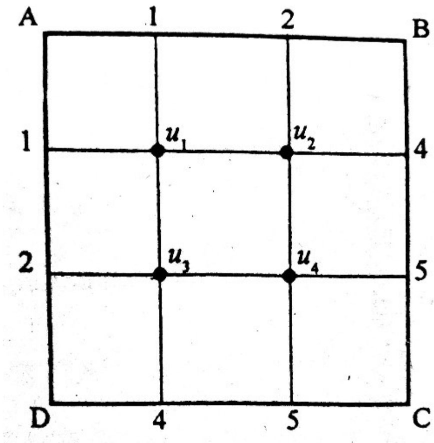
[5+5] **CO1**

Q10. B

Fit a polynomial of degree three, which takes the following values, by Newton backward interpolation formula, and find $y(5.5)$.

x	3	4	5	6
y	6	24	60	120

[8+2] **CO2**

Q11. A	Solve the elliptic equation $u_{xx}+u_{yy}=0$ for the following square mesh with boundary values as shown in the figure by Liebmann's iteration process. Perform five iterations.  **OR** Solve the Poisson's equation $u_{xx}+u_{yy}=8x^2y^2$ over the square mesh with sides $x=0, y=0, x=3, y=3$ with $u=0$ on the boundary and mesh length 1. Perform three iterations by Gauss Seidal method to solve the linear equations in u.	**[10]**	**CO6**
Q11. B	Solve $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t)=0, u(4,t)=0, u(x,0)=x(4-x)$ taking $h=1$ and employing Bender-Schmidt method. Continue the solution through five time steps. **OR** Using Crank-Nicholson's method, solve $u_{xx}=u_t, 0<x<5, t>0$, given that $u(x,0)=20, u(0,t)=0, u(5,t)=100$. Compute u for two steps in t direction taking $h=1$.	**[10]**	**CO6**