Roll No: --------------------------

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme: B.Tech. (APE-UP, FSE) **Semester – IV**
Course Name: Applied Numerical Methods **Max. Marks : 100**
Course Code: MATH-2002 **Duration : 3 Hrs**
No. of page/s: 02

Instructions:
Attempt all questions from **Section A** (each carrying 5 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

SECTION A
(Attempt all questions)

1.	Round off the number 25.9855 to 2 decimal places and compute the relative error in your answer.	**[5]**	**CO1**
2.	Derive Newton-Raphson iteration scheme to find the reciprocal of a positive number α.	**[5]**	**CO1**
3.	Let $f(x)$ be a polynomial of unknown degree satisfying the points $(0,2),(1,7),(2,13)$ and $(3,16)$. If all the fourth divided differences of $f(x)$ are $-\frac{1}{6}$ then find $f(4)$.	**[5]**	**CO2**
4.	Find the missing terms in the following table $\begin{array}{|c|c|c|c|c|c|c|c|} \hline x: & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline f(x): & 0 & 0 & 4 & ? & 48 & ? & 180 \\ \hline \end{array}$ if the area bounded by the cubic curve $f(x), x=0, x=6$ and x- axis is 252 sq. units.	**[5]**	**CO3**

SECTION B
(Q5-Q8 are compulsory and Q9 has internal choice)

5.	Suppose $p(x)$ is a polynomial of degree 2 that approximates the function 2^x for the points $x=0,1\wedge 2$. Find the absolute error in $p(3)$.	**[8]**	**CO2**
6.	Consider a function $f(x)=\lfloor x\rfloor+\lfloor x \rceil$, where $\lfloor x\rfloor$ denotes the greatest integer function which returns the largest integer less than or equal to x and $\lfloor x \rceil$ denotes the round off function which returns the nearest integer to x. Evaluate the integral $\int_1^3 f(x)dx$ by dividing the range of integration $[1,3]$ into 8 equal parts. Also compute the absolute error in the calculated value.	**[8]**	**CO3**
7.	Consider an initial value problem (IVP):	**[8]**	**CO5**

	$$\frac{dy}{dx}=f(x,y),y(x_0)=y_0$$ Suppose $f(x,y)=g(x)$ and $y(x_1=x_0+h)$is calculated using Runge-Kutta method of fourth order. Show that this method eventually reduces to Simpson's rule of numerical integration for $f(x,y)$ with step-size $\frac{h}{2}$.		
8.	Use Taylor's series method to obtain $y(1.1)$ correct to 3decimal places, if given that $y'=yx, y(1)=0$.	**[8]**	**CO5**
9.	Given the diffusion equation $u_t=u_{xx}$ with $u(0,t)=0;u(1,t)=0$ and $u(x,0)=\sin\alpha x$ such that $u(x,0)$has zeros at integer values of x only. Apply Bender-Schmidt method to solve for five time steps taking $h=0.25$. **OR** Solve $u_t=u_{xx}$ with $u(x,0)=u(0,t)=0$ and $u(1,t)=\lim_{\alpha\to 0}\left(\frac{e^{\alpha t}-1}{\sinh\alpha\ \cosh\alpha}\right)$. Compute u for $t=1/8$ in two time steps, using Crank-Nicolson's method.	**[8]**	**CO6**
	SECTION C **(Q10 has internal choice and Q11 is compulsory)**		
10.	Suppose $k$is non-prime and the matrix $A=\begin{bmatrix}1&1&k\\2&k&2\\1&3&2\end{bmatrix}$ is such that $det(A)=-1$. Consider the unique decomposition $A=LU$, where $$L=\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix} \text{ and } U=\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}.$$ Let $X\in R^3\wedge b=[1,1,1]^t$. Find the solution of the system $AX=b$ where $X=[x,y,z]^t$. **OR** Suppose $k$is positive and the matrix $A=\begin{bmatrix}1&1&1\\1&2&k\\1&k&3\end{bmatrix}$ is such that $det(A)=1$. Consider the unique decomposition $A=LU$, where $L=\begin{bmatrix}l_{11}&0&0\\l_{21}&l_{22}&0\\l_{31}&l_{32}&l_{33}\end{bmatrix}$ and $U=L^T$, where L^T denotes the transpose matrix of L. Let $X\in R^3\wedge b=[1,1,3]^t$. Find the solution of the system $AX=b$ where $X=[x,y,z]^t$.	**[20]**	**CO4**

11.	Consider an IVP: $$\frac{dy}{dx}=y+(2x-1)e^{x^2},\ y(0)=1$$ Find the value of $y(1)$ using Euler's method with $h=\frac{1}{4}$. Also obtain the actual solution of the given IVP and compute the absolute error in the calculated value.	**[20]**	**CO5**

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SECTION A
(Attempt all questions)

1.	Compute the relative error in your answer if the number 5.9995 is truncated to 2 decimal places.	**[5]**	**CO1**
2.	Let $f(x)=(x-1)^9$and $x_n=1+\frac{1}{n^2}, n=1,2,3,\ldots$ be the sequence of approximations converging to the root $\alpha=1$. Find $n$for which $\lvert\alpha-x_n\rvert<10^{-4}$.	**[5]**	**CO1**
3.	Suppose $p(x)$ is a polynomial of degree 2 such that $p(x)=e^x$ at the points $x=0,1\wedge 2$. Calculate $p(3)-e^3$.	**[5]**	**CO2**
4.	Suppose $f(x)$ is a cubic polynomial which bounds an area of 324 sq. units between $x=0$ and $x=6$ above $x-¿$axis. If $f(x)$ passes through the points $(0,0),(1,1),(2,8),(3,a),(4,64),(5,b)$ and $(6,216)$ find the values of a and b.	**[5]**	**CO3**

SECTION B
(Q5-Q8 are compulsory and Q9 has internal choice)

5.	Suppose all the fourth divided differences of the polynomial $f(x)$ are $-\frac{1}{6}$ and $f(x)$ satisfies the data:	**[8]**	**CO2**

<table>
<tr><td></td><td>
<table>
<tr><td>x:</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>f(x):</td><td>2</td><td>7</td><td>13</td><td>16</td></tr>
</table>
Find $f(x)$.</td><td></td><td></td></tr>
<tr><td>6.</td><td>Evaluate the integral

$$I=\int_{2}^{4}[x]-(x)\ \mathrm{dx}$$

by considering 9 ordinates, where $[x]$ denotes the greatest integer function which returns the largest integer less than or equal to x and (x) denotes the round off function which returns the nearest integer to x. Also compute the absolute error in the calculated value.</td><td>[8]</td><td>CO3</td></tr>
<tr><td>7.</td><td>Suppose $y(x_0)=y_0$ and Runge-Kutta method (IV order) is applied on $y=\int_{0}^{x}g(t)dt$ to calculate $y(x_1=x_0+h)$. Show that this method eventually reduces to Simpson's rule of numerical integration for $g(x)$ with step-size $\frac{h}{2}$.</td><td>[8]</td><td>CO5</td></tr>
<tr><td>8.</td><td>Use Taylor's series method to obtain $y(1.1)$ correct to 3decimal places, if given that $y'(x)=y(x)$ with $y=1$ at $x=0$.</td><td>[8]</td><td>CO5</td></tr>
<tr><td>9.</td><td>Given the diffusion equation $u_t=u_{xx}$ with $u(0,t)=0; u(1,t)=1$ and $u(x,0)=\sin\alpha x$ such that $u(x,0)$has zeros at even integer values of x only. Apply Bender-Schmidt method to solve for five time steps taking $h=0.25$.

OR

Solve $u_t=u_{xx}$ with $u(x,0)=u(0,t)=0$ and $u(1,t)=\lim_{\alpha\to 0}\left(\frac{e^{\alpha t}-e^{-\alpha t}+\tan\alpha t}{\sinh(3\alpha)}\right)$.
Compute u for $t=1/8$ in two time steps, using Crank-Nicolson's method.</td><td>[8]</td><td>CO6</td></tr>
<tr><td colspan="4">SECTION C
(Q10 has internal choice and Q11 is compulsory)</td></tr>
<tr><td>10.</td><td>Consider the matrix $A=\begin{bmatrix}1&1&k\\2&k&2\\1&3&2\end{bmatrix}$ such that $det(A)+1=0$ where $k$is non-prime. Use Doolittle's method to solve the system $AX=b$ where $X=[x,y,z]^t$ and $b=[3,5,6]^t$.

OR

Suppose $k$is positive and the matrix $A=\begin{bmatrix}1&1&1\\1&2&k\\1&k&3\end{bmatrix}$ is such that $det(A)=1$. Consider the unique decomposition $A=LU$, where</td><td>[20]</td><td>CO4</td></tr>
</table>

	$L=\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ and $U=L^T$, where L^T denotes the transpose matrix of L. Let $X \in R^3 \wedge b=[3,5,6]^t$. Find the solution of the system $AX=b$ where $X=[x,y,z]^t$.		
11.	Consider an IVP: $y'(x)=\sin x+y(x),\ y(0)=1$ Find the value of $y(1)$ using Euler's method with $h=\frac{1}{4}$. Also obtain the actual solution of the given IVP and compute the absolute error in the calculated value.	**[20]**	**CO5**