Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2020

Course: Real Analysis **Semester: II**
Course Code: MATH 1018 **Time: 03 hrs.**
Programme: B.Sc H(Mathematics) **Max. Marks: 100**
Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 20 multiple choice questions and 5 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 10:00 AM to 1:00 PM on 6th July 2020. The due time for PART A is 1:00 PM on 6th July 2020. After the due time, the PART A will not be available.

S. No.		Marks	CO
Q1 (i)	The geometric series $1 + x + x^2 + x^3 + \cdots$ (more than one answer may be correct) A. Converges if $-1 < x < 1$ B. Diverges if $x \geq 1$ C. Oscillates finitely if $x = -1$ D. Oscillates infinitely if $x < -1$	**3**	**CO5**
Q1 (ii)	The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p}$ ... (more than one answer may be correct) A. Converges if $p > 1$ B. Diverges if $p \leq 1$ C. Converges if $p < 1$ D. Diverges if $p \geq 1$	**3**	**CO5**
Q1 (iii)	Using D 'Alembert's ratio test the series $\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \cdots$ (more than one answer may be correct) A. Convergent if $x < 1$ B. Divergent if $x > 1$ C. Convergent if $x = 1$ D. Divergent if $x = 1$	**3**	**CO4**
Q1 (iv)	Consider the series $u_n = \begin{cases} 2^{-n} & if\ n\ is\ odd \\ 2^{-n+2} & if\ n\ is\ even \end{cases}$ then (more than one answer may be correct) A. Using Cauchy's root test $\sum u_n$ is convergent	**3**	**CO4**

	B. D' Alembert's ratio test fails C. Using Cauchy's root test $\sum u_n$ is divergent D. Using D' Alembert's ratio test $\sum u_n$ is convergent		
Q1 (v)	The Sequence whose nth term is $\frac{2n-7}{3n+2}$ (more than one answer may be correct) A. Is monotonically increasing B. Bounded C. Tends to limit $\frac{2}{3}$ D. Is monotonically decreasing	**3**	**CO3**
Q1 (vi)	Which of the following is correct (more than one answer may be correct) A. The set of real numbers is not countable B. The set of all rational numbers is countable C. The set of irrational numbers is countable D. The set of real numbers is countable	**2**	**CO4**
Q1 (vii)	The Sequence whose nth term is $\frac{n}{n^2+1}$ (more than one answer may be correct) A. Is monotonically increasing B. Bounded C. Tends to limit 0 D. Is monotonically decreasing The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ is A. Conditionally convergent B. Absolute convergent C. Divergent D. Convergent	**2**	**CO1**
Q1 (viii)	If $\sum u_n$ is a series of positive terms such that $\lim_{n\to\infty}(u_n)^{1/n}=l$ then (More than one answer may be correct) A. $\sum u_n$ is convergent if $l<1$ B. $\sum u_n$ is divergent if $l>1$ C. $\sum u_n$ may converge or diverge if $l=1$ D. $\lim_{n\to\infty}(u_n)^{1/n}=\infty$, then $\sum u_n$ is divergent.	**2**	**CO5**
Q1 (ix)	If $\sum u_n$ is a series of positive terms such that $\lim_{n\to\infty} n\frac{u_n}{u_{n+1}}=l$, then (more than one answer may be correct)	**2**	**CO5**

	A. $\sum u_n$ is convergent if $l > 1$ B. $\sum u_n$ is divergent if $l < 1$ C. $\sum u_n$ is convergent if $l < 1$ D. $\sum u_n$ is divergent if $l > 1$		
Q1 (x)	The set $\left\{\frac{1}{n} : n \in N\right\}$ is an A. Infinite set having only one limit point B. Finite set having only one limit point C. Infinite set having more than one limit point D. Finite set having more than one limit point	**2**	**CO5**
Q1 (xi)	The series $\sum(-1)^{n-1}u_n = u_1 - u_2 + u_3 - u_4 + \cdots \quad (u_n > 0 \ \ \forall n)$ converges if (More than one answer may be correct) A. $u_n \geq u_{n+1} \ \forall n$ B. $\lim_{n\to\infty} u_n = 0$ C. $u_n \leq u_{n+1} \ \forall n$ D. $\lim_{n\to\infty} u_n = 1$	**2**	**CO5**
Q1 (xii)	According to Bolzano-Weierstrass theorem: Every ________ and _______ subset of R has a limit point. A. Infinite , Bounded B. Finite , Bounded C. Infinite , Unbounded D. Finite, Unbounded	**2**	**CO2**
Q1 (xiii)	Using Comparison test the series $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots$ is A. Convergent B. Divergent C. Test fails D. None of these	**2**	**CO3**
Q1 (xiv)	The set $\left\{\frac{1}{n} : n \in N\right\}$ is an A. Infinite set having only one limit point B. Finite set having only one limit point C. Infinite set having more than one limit point	**2**	**CO2**

	D. Finite set having more than one limit point		
Q1 (xv)	The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ is A. Conditionally convergent B. Absolute convergent C. Divergent D. Convergent	**3**	**CO4**
Q1 (xvi)	If $\langle a_n\rangle$ converges to$l$, then the sequence $\langle x_n\rangle$ where $x_n=\frac{a_1+a_2+\cdots a_n}{n}$Converges to A. l B. 0 C. ∞ D. 1	**2**	**CO3**
Q1 (xvii)	How many cluster points does the sequences$\langle n\rangle$, $\langle\frac{1}{n}\rangle$ and $\langle(-1)^n\rangle$ have. A. none, one, two B. one, two, three C. none, one, one D. none, none, one	**2**	**CO3**
Q1 (xviii)	The limit superior and limit inferior of the following sequence $\langle a_n\rangle$ where $a_n=\sin\frac{n\pi}{3}$ A. $\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}$ C. $-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}$ D. $-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$	**2**	**CO5**
Q1 (xix)	The supremum and infimum of the set $\left\{-2,-\frac{3}{2},-\frac{4}{3},-\frac{5}{4},\dots\dots..\right\}$ A. $1,-2$ B. $-1,-2$ C. $-2,1$ D. $0,-1$	**2**	**CO2**
Q1 (xx)	Consider the following statements i. An interval which is closed set ii. An interval which is not a closed set	**2**	**CO1**

	iii. A set which is neither open nor closed Consider the following examples a. $[2,3]$ b. $(2,3)$ c. $[2,3)$ Choose the correct match A. i-a, ii-c, iii-b B. i-b, ii-a, iii-c C. i-a, ii-b, iii-c D. i-c, ii-b, iii-a		
Q1 (xxi)	Using D' Alembert Ratio test the following series $\frac{2!}{3}+\frac{3!}{3^2}+\frac{4!}{3^3}+\cdots$ A. Convergent B. Divergent C. Test Fails D. None of these	**3**	**CO4**
Q1 (xxii)	In the series $\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+\cdots$ then E. By D' Alembert's Ratio test series is convergent F. By D' Alembert's Ratio test series is divergent G. By Raabe's test series is convergent H. By Raabe's test series is divergent.	**3**	**CO4**
Q1 (xxiii)	The series $1^2+2^2+3^2+\cdots$ A. diverges to $-\infty$ B. converges to 1 C. diverges to ∞ D. converges to $\frac{1}{2}$	**2**	**CO2**
Q1 (xxiv)	The series $-1-2-3-\cdots$ A. diverges to $-\infty$ B. converges to 1 C. diverges to ∞ D. Oscillates finitely	**2**	**CO2**
Q1 (xxv)	The series $1-1+1-\cdots$ A. diverges to $-\infty$ B. converges to 1 C. diverges to ∞ D. Oscillates finitely	**2**	**CO2**

PART B			
The link for PART B will be available from 10:00 AM on 6th July 2020 to 10:00 AM on 7th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_BscH_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.			
Q2	Prove that the set of all rational numbers is countable.	**8**	**CO**
Q3	Discuss the convergence of the following series i. $1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \frac{4!}{4^2} + \ldots\ldots$ ii. $1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} \ldots\ldots\ldots..(p > 0)$	**8**	**CO**
Q4	A sequence $\langle a_n \rangle$ is defined as $a_1 = 1,\ a_{n+1} = \frac{4+3a_n}{3+2a_n}, n \geq 1$. Show that the sequence $\langle a_n \rangle$ converges and find its limit.	**8**	**CO**
Q5	Find the limit superior and limit inferior of the following sequence i. $\langle a_n \rangle$ where $a_n = \sin\frac{n\pi}{3}$ ii. $\langle a_n \rangle$ where $a_n = (-1)^n(2^n + 3^n)$	**8**	**CO**
Q6	. Using Integral test, show that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$.	**8**	**CO**