Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2020

Course: PDE and system of ODE — **Semester: IV**

Course Code: MATH 2030 — **Time: 03 hrs.**

Programme: B.Sc.(H) Maths — **Max. Marks: 100**

Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 20 multiple choice questions and 5 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 2:00 PM to 5:00 PM on 8th July 2020. The due time for PART A is 5:00 PM on 8th July 2020. After the due time, the PART A will not be available.

S. No.		Marks	CO
Q1 (i)	For an arbitrary function f, the PDE corresponding to the family $z = f(x^2 - y^2)$ is A. $xz_x + yz_y = 0$ B. $yz_x + xz_y = 0$ C. $z_{xx} = 0$ D. $z_{yy} = 0$	2	CO1
Q1 (ii)	Which of the following is a second order quasilinear PDE? A. $\frac{\partial u}{\partial x}.\frac{\partial^2 u}{\partial x^2} = 0$ B. $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - u = 0$ C. $\frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial x^2} = u$ D. $u\frac{\partial u}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2}\right)^2 = 0$	2	CO1
Q1 (iii)	The characteristic curves for the PDE $x^2u_{xx} - y^2u_{yy} = x^2y^2 + x; x > 0$ A. are branches of a hyperbola B. is an ellipse centered at origin C. are the lines through origin D. is a parabola with eccentricity 1.	2	CO3

Q1 (iv)	Which of the following is FALSE? A. The diffusion equation $u_t = c^2 u_{xx}$ is parabolic for all choices of $c \neq 0$ B. The wave equation $u_{tt} = c^2 u_{xx}$ is hyperbolic for all choices of $c \neq 0$ C. The Laplace equation $u_{xx} + u_{yy} = 0$ is elliptic D. The Poisson equation $u_{xx} + u_{yy} = f(x, y)$ is not elliptic for some choice of $f(x, y)$	**2**	**CO2**
Q1 (v)	For some arbitrary constants c_i $(i = 1,2)$ and for arbitrary functions f and g, the most general solution to the PDE $\frac{\partial^2 u}{\partial x^2} = 0$ is A. $u(x,y) = x(f(y) + c_1)$ B. $u(x,y) = c_1 x + c_2$ C. $u(x,y) = x + f(y)$ D. $u(x,y) = xf(y) + g(y)$	**2**	**CO2**
Q1 (vi)	The homogeneous solution u_h (for $F(x,y) = 0$) to the PDE $\frac{\partial^2 u}{\partial x \partial y} = F(x,y)$ is (where f and g are arbitrary functions) A. $u_h(x,y) = f(-y) + g(-x)$ B. $u_h(x,y) = f(y)$ C. $u_h(x,y) = f(y)g(x)$ D. $u_h(x,y) = xyf(y)g(x)$	**2**	**CO1**
Q1 (vii)	Which of the following is a possible solution of the heat equation ? A. $u(x,t) = e^{-t} \sin 2x$ B. $u(x,t) = e^{-t} (\sin x + \cos x)$ C. $u(x,t) = e^{4t} (\sin 2x + \cos 2x)$ D. $u(x,t) = e^{t} \sin x$	**2**	**CO2**
Q1 (viii)	Which of the following is a possible solution of the vibrating string problem for a string of length L with fixed ends and zero initial velocity? A. $u(x,t) = \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi t}{L}\right)$ B. $u(x,t) = \sin\left(\frac{\pi x}{2L}\right) \cos(\pi t)$ C. $u(x,t) = \sin\left(\frac{\pi x}{L}\right) \cos(\pi t L)$ D. $u(x,t) = \sin\left(\frac{\pi x}{2L}\right) \cos\left(\frac{\pi t}{2L}\right)$	**2**	**CO2**
Q1 (ix)	Let $\phi(t)$ be the fundamental matrix for the solution $X(t)$ of the system $\frac{dX}{dt} = AX$. Then for some column vector where $\boldsymbol{c}$ A. $X(t) = \phi(t) + \boldsymbol{c}$ B. $X(t) = \boldsymbol{c}\phi(t)$ C. $X(t) = \phi(t)\boldsymbol{c}$ D. $X(t) = \boldsymbol{c} + \phi(t)$	**2**	**CO4**

Q1 (x)	Let $\phi(t)$ denote the fundamental matrix for the homogeneous solution $X_h(t)$ of the system $\frac{dX}{dt} = AX + F(t)$. Then the particular solution is A. $X_p(t) = \phi(t)\int \phi^{-1}(u)F(u)du$ B. $X_p(t) = \phi^{-1}(t)\int \phi(u)F(u)du$ C. $X_p(t) = \phi(t) + \int \phi^{-1}(u)F(u)du$ D. $X_p(t) = \phi^{-1}(t) + \int \phi(u)F(u)du$	**2**	**CO4**
Q1 (xi)	Let $X_p(t)$ denote the particular solution of the IVP: $$\frac{dX}{dt} = AX + F(t), X(t_0) = X_0$$ Then which of the following is true? A. $X_p(t) = X_0$ B. $X_p(t) = 1$ C. $X_p(t) = -1$ D. $X_p(t) = 0$	**2**	**CO4**
Q1 (xii)	Which of the following CANNOT be the fundamental matrix for the solution of system $\frac{dX}{dt} = AX$? A. $\begin{pmatrix} e^t & 3e^{-t} \\ 2e^t & e^{-t} \end{pmatrix}$ B. $\begin{pmatrix} e^t & 3e^{-t} \\ 2e^t & 6e^{-t} \end{pmatrix}$ C. $\begin{pmatrix} e^t & -e^{-t} \\ e^t & e^{-t} \end{pmatrix}$ D. $\begin{pmatrix} 3e^t & 6e^{-t} \\ -e^t & 2e^{-t} \end{pmatrix}$	**2**	**CO4**
Q1 (xiii)	Let $u(x,t)$ be the solution of the initial value problem $$u_{tt} - u_{xx} = 0, u(x,0) = x^3, u_t(x,0) = \sin x.$$ Then $u(\pi,\pi)$ is A. $4\pi^3$ B. π^3 C. 0 D. 4	**2**	**CO3**
Q1 (xiv)	The solution of $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) z = 0$ is A. $y - x$ B. $\sin(y - x)$ C. $\sqrt{y - x}$ D. All of the above	**2**	**CO1**
Q1 (xv)	For the IVP $u_{tt} = 4u_{xx}, t > 0, -\infty < x < \infty$ satisfying the conditions $u(x,0) = x, u_t(x,0) = 0$, the D-Alembert's solution is A. $u(x,t) = 0$ B. $u(x,t) = x^2$ C. $u(x,t) = x$ D. $u(x,t) = xt$	**2**	**CO2**

Q1 (xvi)	Let $u(x,t)$ be the solution of the IVP $u_{tt} = c^2 u_{xx}, t > 0, -\infty < x < \infty \; u(x,0) = f(x), u_t(x,0) = g(x),$ where both $f(x)$ and $g(x)$ are odd functions. Then A. $u(0,1) = 0$ B. $u(0,1) = f(c)$ C. $u(0,1) = \int_{-c}^{c} g(x)dx$ D. $u(0,1) = f(c) + \int_{-c}^{c} g(x)dx$	**2**	**CO2**
Q1 (xvii)	The solution to the IVP $u_t + 2u_x = 0$ with $u_t(x,0) = x$ by Lagrange's method A. cannot be determined B. is unique C. is not unique D. is a family of straight lines	**2**	**CO1**
Q1 (xviii)	The restriction that must be placed on k so that the Cauchy problem $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u - 1, u(ky,y) = 2y$ can be solved is A. $k = 0$ B. $k \neq 0$ C. $k = -\frac{1}{2}$ D. $k \neq -\frac{1}{2}$	**2**	**CO3**
Q1 (xix)	The IVP $\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = 0, u(x,0) = \sin x, u_t(x,0) = x$ has A. a unique solution $u(x,t) = \sin(x - 3t)$ B. no solution C. infinitely many solutions D. a solution $u(x,t) = \sin(3x - t)$	**2**	**CO3**
Q1 (xx)	Let $u(x,t)$ be the unique solution of $u_{tt} - u_{xx} = 0, x \in \mathbb{R}, t > 0, u(x,0) = f(x), u_t(x,0) = 0.$ where $f(x) = x(1-x) \; \forall x \in [0,1]$ and $f(x+1) = f(x) \; \forall x \in \mathbb{R}$. Then $u\left(\frac{1}{2}, \frac{5}{4}\right)$ is A. $\frac{1}{8}$ B. $\frac{1}{16}$ C. $\frac{3}{16}$ D. $\frac{5}{16}$	**2**	**CO3**

Q1 (xxi)	Let $(x(t), y(t))$ satisfy for $t > 0$ the system $\frac{dx}{dt} = -x + y, \frac{dy}{dt} = -y, x(0) = y(0) = 1$. Then $x(t)$ is equal to A. $e^{-t} + ty(t)$ B. $y(t)$ C. $e^{-t}(1+t)$ D. $-y(t)$	**4**	**CO1**
Q1 (xxii)	Consider the system $\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x(t) + y(t) \\ -y(t) \end{bmatrix}$. Then which of the following is NOT the solution? A. $\begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}$ and $\begin{bmatrix} e^{t} \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} e^{t} \\ 0 \end{bmatrix}$ and $\begin{bmatrix} \cosh t \\ e^{-t} \end{bmatrix}$ C. $\begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ and $\begin{bmatrix} \sinh t \\ e^{-t} \end{bmatrix}$ D. $\begin{bmatrix} e^{t} \\ 0 \end{bmatrix}$ and $\begin{bmatrix} e^{t} - \frac{1}{2}e^{-t} \\ e^{-t} \end{bmatrix}$	**4**	**CO4**
Q1 (xxiii)	Let $u(x,t)$ be the solution of $u_{xx} - u_{tt} = 0\,, u(x,0) = f(x), u_t(x,0) = 0\,, x \in \mathbb{R}, t > 0$. If $f(2) = 4, f(0) = 0$ and $u(2,2) = 8$, then the value of $u(3,1)$ is A. 12 B. 6 C. 2 D. 0	**4**	**CO2**
Q1 (xxiv)	The particular solution of $\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y}\right) z = x$ is A. $\frac{x^2}{2}$ B. $-y\left(x - \frac{y}{2}\right)$ C. $-y\left(x + \frac{y}{2}\right)$ D. $-y\left(y + \frac{x}{2}\right)$	**4**	**CO1**
Q1 (xxv)	Let $F(u,v) = 0$ be the general solution of $(2xy - 1)\frac{\partial z}{\partial x} + (z - 2x^2)\frac{\partial z}{\partial y} = 2(x - yz)$. Then A. $u = x^2 + y^2 + z, v = xz + y$ B. $u = x^2 + y^2 - z, v = xz - y$ C. $u = x^2 - y^2 + z, v = yz + x$ D. $u = x^2 + y^2 - z, v = yz - x$	**4**	**CO1**

PART B			
The link for PART B will be available from 2:00 PM on 8th July 2020 to 5:00 PM on 9th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.			
Q2	Consider the first order PDE: $$x\frac{\partial u}{\partial x} + (1+y)\frac{\partial u}{\partial y} = x(1+y) + xu$$ a. Find the general solution of the given PDE. b. Assume the initial condition is of the form $u(x, 6x-1) = \phi(x)$. Find the necessary and sufficient condition on $\phi(x)$ that guarantees the existence of a solution. Solve the problem for the appropriate $\phi(x)$.	**8**	**CO1**
Q3	Solve the following IBVP using D' Alembert's method: $$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, \qquad x \in (-\infty, \infty), t > 0$$ $$u(x,0) = \begin{cases} 2, & \lvert x\rvert < 3 \\ 0, & \lvert x\rvert \geq 3 \end{cases}$$ $$\frac{\partial u}{\partial t}(x,0) = 0$$	**8**	**CO2**
Q4	Consider the following heat conduction problem: $$\frac{\partial^2 u}{\partial x^2} - 2u = \frac{1}{3}\frac{\partial u}{\partial t} \text{ where } 0 < x < 2,\ t > 0$$ $$u(0,t) = u(2,t) = 0, u(x,0) = x$$ Find the solution of the form $u(x,t) = \phi(x)T(t)$ for the given problem.	**8**	**CO2**
Q5	Solve the boundary value problem: $$u_{xx} = u_{tt} + u,\ 0 < x < \pi, t > 0$$ with the boundary conditions $u_x(0,t) = 0$ and $u_x(\pi,t) = 0$ for $t > 0$ and the initial conditions $u(x,0) = 0$ and $u_t(x,0) = f(x)$ for $0 < x < \pi$.	**8**	**CO3**
Q6	Suppose $\phi(t)$ is the fundamental matrix corresponding to the coefficient matrix A in the IVP: $$\frac{dX}{dt} = AX + F(t), \qquad X(t_0) = X_0, t_0 \in [a,b]$$ Find the particular solution of the IVP:	**8**	**CO4**