Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
Online End Semester Examination, December 2020

Course: **Probability Theory & Statistics** **Semester: V**
Program: **B.Sc. (Hons.) Mathematics** **Time: 03 hrs.**
Course Code: **MATH 3013** **Max. Marks: 100**
Instructions: All questions are compulsory.

SECTION A (Each question carries 5 marks)

S. No.		**Marks**
Q1	Let the first four moments of a distribution about the value 5 be 2, 20, 40 and 50. Then the mean of the distribution is A. 4 B. 7 C. 3 D. 2	**CO1**
Q2	If X represents the outcome, when a fair die is tossed, then the moment generating function of X is given by ______	**CO1**
Q3	Consider the following distribution function $f(x) = \lambda e^{-x/t}\ , \quad 0 \leq x < \infty, \lambda > 0$ Then the third moment about origin is A. $3/\lambda^3$ B. $6/\lambda^3$ C. $9/\lambda^3$ D. $12/\lambda^3$	**CO1**
Q4	A random variable X has an exponential distribution with probability density function given by $f(x) = 3e^{-3x}, for\ x > 0$ and zero elsewhere then the probability that X is not less than 4 is _______	**CO2**
Q5	If $f\ (x, y) = k\ (1 - x - y), 0 < x, y < \frac{1}{2}$, is a joint density function then k = _____	**CO3**
Q6	The transition probability matrix of a Markov chain $\{X_n\}, n = 1, 2, 3$ .... Having three states 1, 2 and 3 is $p = \begin{matrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{matrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$ then $P\ \{X_2 = 3\} =$ ______	**CO5**

<table>
<tr><td colspan="3">SECTION B (Each question carries 10 marks)</td></tr>
<tr><td>Q7</td><td>In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of 10,000 packets.</td><td>CO2</td></tr>
<tr><td>Q8</td><td>The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}$; $0 \le x \le 2, 0 \le y \le 1$. Compute $P\ (X > 1), P\left(Y < \frac{1}{2}\right) and\ P\ (X > 1/ Y < \frac{1}{2})$.</td><td>CO3</td></tr>
<tr><td>Q9</td><td>A fair dice is 720 times. Use Chebyshev’s inequality to find a lower bound for the probability of getting 100 to 140 sixes.</td><td>CO4</td></tr>
<tr><td>Q10</td><td>Examine if the weak law of large numbers holds for the sequence $\{X_p\}$ of independent identically distributed random variables with $P[X_k = (-1)^{k-1}.k] = \frac{6}{\pi^2 k^2}, k = 1, 2, \dots; p = 1, 2, \dots..$</td><td>CO4</td></tr>
<tr><td>Q11</td><td>The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200 hours and standard deviation 250 hours. Find the probability, using central limit theorem that the average lifetime of 60 bulbs exceeds 1250 hours.
OR
If $X_1, X_2, X_3, \dots\dots\dots\dots X_n$ are Poisson variate with parameter lambda is equal to 2, Use the central limit theorem to estimate $P\ (120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + X_3 \dots\dots\dots\dots + X_n\ and\ n = 75$.</td><td>CO4</td></tr>
<tr><td colspan="3">SECTION-C (This question carries 20 marks)</td></tr>
<tr><td>Q 12</td><td>Calculate the coefficient of correlation and obtain the lines of regression for the following data:
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td><td>16</td><td>15</td></tr>
</table>
OR
Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X, Y). Also find all the marginal and conditional probability distributions</td><td>CO3</td></tr>
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