Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, June 2021

Programme Name: M.Tech. Automation & Robotics Engineering **Semester : II**
Course Name : Optimization Techniques **Time : 03 hrs**
Course Code : ECEG 7010 **Max. Marks : 100**
Nos. of page(s) : 03

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Instructions: Attempt all questions. Scientific calculators are allowed for the examination.

SECTION A
(All the questions are compulsory; Each questions carries 5 marks)

S. No.		CO
Q1.	The optimal value of the objective function of the linear programming problem $\min_{x_1,x_2\geq 0} 3x_1+2x_2$ subject to $5x_1+x_2\geq 10, x_1+x_2\geq 6, x_1+4x_2\geq 12$ is A. 20 B. 36 C. 13 D. 16	**CO1**
Q2.	The dual objective function of the linear programming problem $\min_{x_1,x_2\geq 0} 3x_1+2x_2$ subject to $5x_1+x_2\geq 10, x_1+x_2\geq 6, x_1+4x_2\geq 12$ is A. max $3v_1+2v_2$ B. max $10v_1+6v_2+12v_3$ C. min $10v_1+6v_2+12v_3$ D. max $5v_1+3v_2$	**CO1**
Q3.	The function $f(x_1,x_2,x_3)=\frac{1}{2}(x_1^2+x_2^2+x_3^2)-2x_1-3x_2-4x_3+11$ has A. maximum value at (2,3,4) B. maximum value at (4,6,8) C. minimum value at (2,3,4) D. minimum value at (4,6,8)	**CO3**
Q4.	The following function $f(x_1,x_2)=x_1^2-2x_1x_2+2x_2^2-4x_1$ is convex A. for all $x_1,x_2\in\mathbb{R}$ B. for $x_1,x_2\notin\mathbb{R}$ C. for $x_1\neq 0$ D. for $x_2\neq 0$.	**CO3**
Q5.	For what values of b the matrix $\begin{bmatrix}2 & -1 & b\\ -1 & 2 & -1\\ b & -1 & 2\end{bmatrix}$ is positive semidefinite? A. $b\leq -1$ B. $b\geq 2$ C. $-1\leq b\leq 2$ D. $b\in\mathbb{R}$	**CO3**
Q6.	The steepest descent direction to minimize the function $f(x_1,x_2,x_3)=2x_1x_3^2+x_1x_2x_3$ at the starting point $(1,-1,-1)$ is A. $\begin{pmatrix}-3\\1\\5\end{pmatrix}$ B. $\begin{pmatrix}3\\-1\\5\end{pmatrix}$ C. $\begin{pmatrix}3\\1\\-5\end{pmatrix}$ D. $\begin{pmatrix}3\\1\\5\end{pmatrix}$	**CO3**

SECTION B

(Q7-Q10 are compulsory, and Q11 has internal choice; Each question carries 10 marks)

Q7. Using Big-M method show that the following linear programming problem has no feasible solution. **CO1**

$$\max z = -2x_1 + x_2 + 3x_3$$
$$\text{subject to} \quad x_1 - 2x_2 + 3x_3 = 2$$
$$3x_1 + 2x_2 + 4x_3 = 1$$
$$x_1, x_2, x_3 \geq 0.$$

Q8. A transportation company ships truckloads of grain from three silos to four mills. The supply (in truckloads), the demand (also in truckloads) and the unit transportation costs (in hundreds of rupees) per truckloads on the different routes are summarized below. The company has an initial shipping schedule: $x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8$ and $x_{34} = 10$. Check optimality of the schedule by modified distribution (MODI) method. If not optimal, find the optimal shipping schedule between the silos and mills, and minimum transportation cost. **CO2**

Mills

		I	II	III	IV	Supply
Silos	A	19	30	50	10	7
	B	70	30	40	60	9
	C	40	8	70	20	18
	Demands	5	8	7	14	

Q9. The head of the department of an IT sector has five jobs A, B, C, D, E and five sub-ordinates X, Y, Z, W, U. The following table gives the number of minutes each man would take to perform each job. How would be the jobs be allocated, one per sub-ordinate, to minimize the total time? **CO2**

	X	Y	Z	W	U
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Q10. A medical representative has to visit five stations A, B, C, D and E, starting from A. The cost of going one station to another are given in the following table. He does not want to visit any station twice before completing his tour of all stations and wishes to return to the starting station. Solve the travelling salesman problem to determine the route he should select so that total travelling cost is minimum. **CO2**

To

		A	B	C	D	E
From	A	--	2	5	7	1
	B	6	--	3	8	2
	C	8	7	--	4	7
	D	12	4	6	--	5
	E	1	3	2	8	--

Q11.	Solve the following linear problem by dynamic programming technique. $$\max_{x_1,x_2\geq 0} 8x_1 + 7x_2$$ subject to $2x_1 + x_2 \leq 8$ $5x_1 + 2x_2 \leq 15$ **OR** Solve the following non-linear problem by dynamic programming technique. $$\max_{x_1,x_2,x_3\geq 0} x_1^2 x_2 x_3$$ subject to $x_1 + 2x_2 + x_3 = 15$ $x_1, x_2, x_3 \geq 0.$	**CO4**
SECTION C **Q12a. and Q12b. both have internal choices; Each question carries 10 marks**		
Q12.	a. Compute Karush-Kuhn-Tucker (KKT) optimality conditions for the following convex programming problem. $$\min_{x_1,x_2\geq 0} -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$$ subject to $2x_1 + x_2 \leq 6$ $x_1 - 4x_2 \leq 0.$ **OR** Using Lagrange multiplier method solve the following constrained optimization problem. $$\min_{x_1,x_2\geq 0} x_1^2 - x_1x_2 + x_2^2$$ subject to $x_1^2 + x_2^2 = 1.$ b. Use Fibonacci search method to minimize the function $f(x) = -\frac{1}{(x-1)^2}\left(\ln x - 2\frac{x-1}{x+1}\right)$ in the range $[1.5,4.5]$. Reduce the size of the interval minimum $\frac{1}{5}$ of the original. **OR** Apply Steepest descent method to minimize the function $f(x_1, x_2) = 4x_1^2 - 4x_1x_2 + 2x_2^2$ with initial point $x_0 = (2,3)$. Perform iterations until $\lvert\nabla f\rvert < \begin{pmatrix}1\\1\end{pmatrix}$.	**CO3**

*****End*****