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| **Name:**<br>**Enrolment No:** | UPES<br>UNIVERSITY WITH A PURPOSE |

**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**
**Online End Semester Examination, MAY 2021**

**Course: Mathematical Physics III** — **Semester: IV**
**Program: B.Sc. (Physics)** — **Time 03 hrs.**
**Course code: PHYS 2004** — **Max. Marks: 100**

**SECTION A**

1. **Each Question will carry 5 Marks**
2. **Instruction: Complete the statement / Select the correct answer(s)**

| S. No. | **Question** | **CO** |
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| Q 1 | If $U(t-a)$ is a unit step function, then $L[U(t-a)]$= | **CO2** |
| Q2 | $L^{-1}\left[\frac{1}{s^n}\right]$ is possible only when n is or<br>a. Positive integer<br>b. Zero<br>c. Negative integer<br>d. Negative rational | **CO2** |
| Q3 | Find the Laplace transform of $t+t^2+t^3$<br>a. $\frac{1}{s^2}+\frac{2}{s^3}+\frac{6}{s^4}$<br>b. $\frac{1}{s^2}+\frac{2}{s^3}+\frac{3}{s^4}$<br>c. $\frac{1}{s^2}+\frac{1}{s^3}+\frac{1}{s^4}$<br>d. $\frac{1}{s}+\frac{2}{s^2}+\frac{3}{s^3}$ | **CO2** |
| Q4 | The value of Dirac delta function is ..... only for a short time, otherwise it will be…….. | **CO1** |
| Q5 | A three dimensional general wave equation propagating in space with velocity *v* can be represented by the equation<br>a. $\frac{\partial^2 Y}{\partial t^2}=v^2\left(\frac{\partial^2 Y}{\partial x^2}+\frac{\partial^2 Y}{\partial y^2}+\frac{\partial^2 Y}{\partial z^2}\right)$<br>b. $v^2\frac{\partial^2 Y}{\partial t^2}=\left(\frac{\partial^2 Y}{\partial x^2}+\frac{\partial^2 Y}{\partial y^2}+\frac{\partial^2 Y}{\partial z^2}\right)$<br>c. $v^2\frac{\partial Y}{\partial t}=\left(\frac{\partial^2 Y}{\partial x^2}+\frac{\partial^2 Y}{\partial y^2}+\frac{\partial^2 Y}{\partial z^2}\right)$<br>d. $\frac{\partial Y}{\partial t}=v^2\left(\frac{\partial^2 Y}{\partial x^2}+\frac{\partial^2 Y}{\partial y^2}+\frac{\partial^2 Y}{\partial z^2}\right)$ | **CO3** |

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| Q6 | An analytic function within a closed contour can be expanded by ……series while, if the function is analytic with in the closed ring bounded by two concentric circles centered at same point expanded by…….series. | **CO1** |
| | **SECTION B**<br>1. **Each question will carry 10marks**<br>2. **Instruction: Write short / brief notes** | |
| Q 7 | (a)State the Cauchy residue theorem for multiply connected region.<br>(b) Evaluate $\oint_c \frac{z^2}{(z-1)^2(z-2)}dz$ ; where c is $\|z\| = 1.5$ | **CO1** |
| Q 8 | If $z = \cos\theta + i\sin\theta$, prove that<br>$\frac{1+z}{1-z} = i\cot\frac{\theta}{2}$ | **CO1** |
| Q 9 | Evaluate<br>(a) $\int_0^\infty e^{-t}t^3 sint\, dt$ (b) $L^{-1}\left[\frac{s^2+2s-3}{s(s-3)(s+2)}\right]$ | **CO2** |
| Q 10 | Find the Fourier transform of $e^{-\frac{r^2}{a^2}}$, where a is a constant and $r = \sqrt{x^2+y^2+z^2}$. | **CO3** |
| Q 11 | Find the Laplace transform of the following function<br>$f(t) = \begin{cases} t & 0 < t \le b \\ 2b - t & b < t < 2b \end{cases}$ where *2b* being the period of $f(t)$.<br>**OR**<br>Find the Fourier transform of<br>$f(x) = \begin{cases} 1-x^2 & if\ \|x\| \le 1 \\ 0 & if\|x\| > 1 \end{cases}$<br>and use it to evaluate $\int_0^\infty \left(\frac{s\cos s - \sin s}{s^3}\right)\cos\frac{s}{2}\, ds$. | **CO2** |
| | **Section C**<br>1. **Each Question carries 20Marks.**<br>2. **Instruction: Write long answer.** | |
| Q12 | Find the equation of motion of an object exhibiting simple harmonic motion with a resistive force (damped harmonic oscillator) and find the solution of the differential equation by the Laplace Transform.<br>**OR**<br>An alternative emf $E = E_0\sin\omega t$ is applied to an inductance L and a capacitance C in series. Show that the current in the circuit is $\frac{E_0\omega}{(n^2-\omega^2)L}(\cos\omega t - \cos nt)$ , where $n^2 = \frac{1}{LC}$ | **CO4** |