	UNIVERSITY OF PETROLEUM AND ENERGY STUDIES **End Semester Examination, December 2021** **Programme Name: B. Sc. (Hons.) Mathematics** **Semester : III** **Course Name : Differential Calculus** **Time : 03 hrs** **Course Code : MATH 1044** **Max. Marks : 100** **Nos. of page(s) : 02**

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q1.	Check the continuity at $x = 1$ for the function $$f(x) = \begin{cases} \frac{\sin x}{x-1}, x < 1 \\ 0, \quad x = 1 \\ \frac{1}{x-1}, x > 1. \end{cases}$$ If it is a point of discontinuity, identify the type of discontinuity.	[3+1]	CO1
Q2.	Verify Lagrange mean value theorem for the function $f(x) = x^3$ in [1,2].	[4]	CO1
Q3.	Find r^{th} order derivative y_r for the function $y = x^n$ when $r < n$, $r = n$ and $r > n$.	[4]	CO2
Q4.	Find the angle of intersection for the curves $y = x^2$ and $x = y^2$ at $(1,1)$.	[4]	CO3
Q5.	Find horizontal and vertical asymptote(s), if exists, for the curve $y = \frac{e^{3x}}{x}$.	[4]	CO4

SECTION B
(Q1-Q3 are compulsory. Q4 have internal choices)

S. No.		Marks	CO
Q1.	If $z = f(x + cy) + g(x - cy)$, show that $z_{yy} = c^2 z_{xx}$.	[10]	CO5
Q2.	Find the length of tangent, normal, sub-tangent and sub-normal of the following curve $x = a(t + \sin t), y = a(1 - \cos t)$ at $t = \frac{\pi}{2}$.	[10]	CO3
Q3.	Obtain all the asymptotes of the curve $$(x^2 - y^2)(x + 2y) + 5(x^2 + y^2) + x + y = 0.$$	[10]	CO4
Q4.	Obtain n^{th} order derivative y_n of the function $y = \frac{1}{(x+2)(2x+3)}$. Or If $y = (Sin^{-1}x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$	[10]	CO2

	SECTION C **(Q1 is compulsory. Q2a and Q2b both have internal choices)**		
Q1	a. Providing necessary information trace the following curve. $x^3 + y^3 = 3axy, a > 0.$ b. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Show that the radius of curvature $\rho = \frac{a^2b^2}{p^3}$, where p is perpendicular distance from origin to the tangent at (x, y).	**[10]** **[10]**	**CO4**
Q2	a. If $u = \ln(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$ Given that $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$. **OR** If $x^x y^y z^z = c$, then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \ln(ex))^{-1}$. b. Consider the function $f(x, y) = \begin{cases} \frac{x^4 + (x^3 - y^3)}{x^2 + y^2}, & if\ (x, y) \neq (0,0) \\ 0, & if\ (x, y) = (0,0). \end{cases}$ Find $f_x(0,0)$ and $f_y(0,0)$. **OR** Let $f = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$. Using Chain rule show that $x^2 \frac{\partial f}{\partial x} + y^2 \frac{\partial f}{\partial y} + z^2 \frac{\partial f}{\partial z} = 0.$	**[10]** **[10]**	**CO5**

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