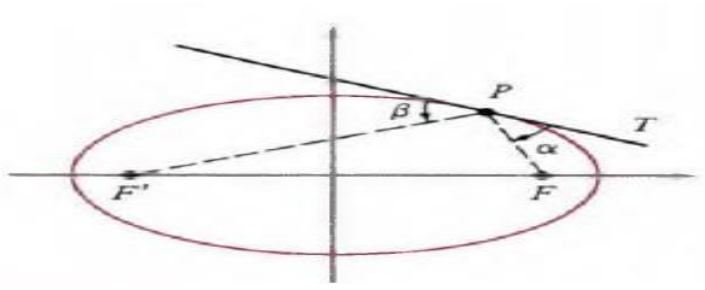
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2021

Programme Name: B. Sc. (Hons.) Mathematics **Semester : V**
Course Name : Analytical Geometry **Time : 03 hrs**
Course Code: MATH 3010D **Max. Marks : 100**

	Section A (All questions are compulsory, each question is of 4 marks)	Marks	CO
1.	Test whether the circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 4x - 6y - 8 = 0$ intersect each other or not.	**4**	**CO1**
2.	Show that the sections of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by the co-ordinate planes are ellipses.	**4**	**CO4**
3.	Discuss the reflection property of the parabola.	**4**	**CO2**
4.	Two parabolas $y^2 = 4a(x - \lambda_1)$ and $x^2 = 4a(y - \lambda_2)$ always touch each other (λ_1, λ_2 being variable parameters). Then their point of contact lies on a…..…(straight line/ circle/ parabola/ hyperbola).	**4**	**CO3**
5.	Show the following statements are true for the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ A. Its section by the *yz*-plane is a hyperbola. B. Its section by the *zx*-plane is a hyperbola.	**4**	**CO4**
	SECTION B **(All questions are compulsory and Q4 has internal choices, each question is of 10 marks)**		
1.	Let *P* be a point on an ellipse with foci *F* and *F'*, and let *T* be the tangent at *P*, as shown in the following figure. If *T* makes angles α and β with the two focal radii *PF* and *PF'*, then prove that $\alpha = \beta$. 	**10**	**CO2**
2.	Identify the graph of $16x^2 - 9y^2 - 64x - 18y + k = 0$ for various values of *k* (*k* is a real number).	**10**	**CO3**
3.	Find the equation of the cone whose vertex is (1, 1, 1) and base is the circle $x^2 + y^2 = 4$, $z = 2$.	**10**	**CO4**
4.	Identify the graph of $16x^2 + 25y^2 = 400$, and find its vertices, foci, eccentricity, and directrices, and sketch its graph. **OR** Classify and sketch the curve $x^2 - 8x - y + 19 = 0$.	**10**	**CO1**

	SECTION C **(All questions are compulsory, each question is of 20 marks and has internal choices)**		
1	A plane passes through a fixed point (*a*, *b*, *c*) and cuts the axes in *A*, *B*, *C*. show that the locus of the centre of the sphere *OABC* (*O* is the origin) is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$	**20**	**CO4**
2	Let *F* be a point, which is inside a given circle but is not the center *C*. Consider a point *P* that moves in such a way as to be equidistant from *F* and the circle. Show that the path of *P* is an ellipse. **OR** Show that the lines tangent to a parabola at the ends of a focal chord (a chord through the focus) intersect on the directrix.	**20**	**CO3**