UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2021

Course: Engineering Mathematics **Semester: I**

Course Code: MATH 1036 **Time: 03 hrs.**

Programme: B.Tech. (All SoCS Batches) **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

Each Question will carry 4 Marks. **(5Qx 4M = 20 Marks)**

		Marks	COs
Q 1	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.	4	CO1
Q 2	Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \log u$ where $\log u = (x^3 + y^3)/(3x + 4y)$.	4	CO2
Q 3	Solve $(D-2)^2 y = (e^x + sin2x)$.	4	CO3
Q 4	A fair coin tossed twice. Let X be the number of heads that are observed. Construct the probability distribution of X.	4	CO4
Q 5	Using Newton-Raphson method, find the real root of $xsinx + cosx = 0$ which is near $x = \pi$ correct to three decimal places.	4	CO5

SECTION B

Each question will carry 10 marks. **(4Qx10M = 40 Marks)**

Q 6	If $y = \text{a} \cos(logx) + b \sin(logx)$, show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.	10	CO2
Q 7	Solve, by the method of variation of parameters, $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.	10	CO3
Q 8	The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that a) at least two will be defective. b) none will be defective.	10	CO4
Q 9	Evaluate $\int_0^1 \frac{1}{1+x}dx$ by dividing the interval of integration into 8 equal parts. Hence find $\log_e 2$ approximately. OR From the following table of half – yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46. Age x : 45 50 55 60 65 Premium y : 114.84 96.16 83.32 74.48 68.48.	10	CO5

SECTION-C

Each Question carries 20 Marks. **(2Qx 20M= 40 Marks)**

Q 10	**a)** Change the order of integration and hence evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dxdy$. **b)** Evaluate $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z} dx\, dy\, dz$. OR **a)** Change the order of integration and hence evaluate $\int_0^a\int_{\sqrt{ax}}^a \frac{y^2dxdy}{\sqrt{y^4-a^2x^2}}$. **b)** Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\, dx\, dy\, dz$.	20	CO2

Q 11	Use Runge – Kutta method of fourth order to find the numerical solution at $x = 0.2$ for $\frac{dy}{dx} = x + y^2$, $y(0) = 1$. Assume step size $h = 0.1$.	**20**	**CO5**