UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Multivariate Calculus **Semester: III**
Program: B.Sc (Hons.) Mathematics **Duration: 03 hrs.**
Course Code: MATH-2029 **Max. Marks : 100**

Instructions:

1. All questions are compulsory.

SECTION A		**(5Q x 4M = 20Marks)**	
S. No.		**Marks**	**COs**
Q1	If u is a homogenous function of degree n in x and y , then show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u.$	4	**CO1**
Q2	State and prove the relation between Beta and Gamma functions.	4	**CO2**
Q3	Evaluate $\int_0^{\pi}\int_0^{a(1-cos\theta)} r^2 sin\theta\, drd\theta$	4	**CO2**
Q4	Show that $\vec{A}=(6xy+z^3)\hat{\imath}+(3x^2-z)\hat{\jmath}+(3xz^2-y)\hat{k}$ is irrotational.	4	**CO3**
Q5	Evaluate the following triple integral $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)\,dxdydz.$	4	**CO2**

SECTION B		**(4Q x 10M = 40Marks)**	
S. No.		**Marks**	**COs**
Q1	Using Lagrange's method of undetermined multipliers, find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.	**10**	**CO1**
Q2	Prove the following identities: (a). $div(curl\,\vec{V})=\nabla.(\nabla\times\vec{V})=0$ (b). $curl(curl\,\vec{V})=grad\,(div\,\vec{V})-\nabla^2\vec{V}$	**10**	**CO3**

Q3	If $\frac{x^2}{2+u}+\frac{y^2}{4+u}+\frac{z^2}{6+u}=1$, prove that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$	**10**	**CO1**
Q4	By changing to cylindrical coordinates, find the volume of the portion of the sphere $x^2+y^2+z^2=a^2$ lying inside the cylinder $x^2+y^2=ay$. **OR** State and prove Liouville's extension of Dirichlet's theorem. Hence, evaluate $\iiint \log(x+y+z)\,dxdydz$, the integral extending over all positive and zero values of x, y, z subject to $x+y+z<1$.	**10**	**CO2**
	SECTION C **(2Q x 20M = 40Marks)**		
S. No.		**Marks**	**COs**
Q1	Find the volume bounded by the solid $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}+\left(\frac{z}{c}\right)^{2/3}=1.$	**20**	**CO2**
Q2	Verify Gauss's divergence theorem for $\vec{F}=2x^2y\hat{\imath}-y^2\hat{\jmath}+4xz^2\hat{k}$ over the region bounded by the cylinder $y^2+z^2=9$ and the plane $x=2$ in the first octant. **OR** Verify Stokes' theorem for $\vec{F}=(y-z)\hat{\imath}+yz\hat{\jmath}-xz\hat{k}$ where S is the region bounded by the planes $x=0, x=1, y=0, y=1, z=0$ and $z=1$ above the $xy-plane$.	**20**	**CO3**