UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Mathematics   Semester: I
Program: BCA   Time: 03 hrs.
Course Code: MATH1037   Max. Marks: 100

Instructions: Answer All the questions. Q9 and Q11 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	**(a)** Find k for which the equation $kx^2 + 4x + (k - 3) = 0$ has equal roots. **(b)** Solve the equation $\frac{1}{2x-5} + \frac{5}{2x-1} = 2$ using quadratic formula.	4	CO1
Q2	Express the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$ as the sum of symmetric and a skew-symmetric matrices.	4	CO2
Q3	In a survey of 80 people, it was found that 35 people read newspaper H, 20 read newspaper T, 15 read the newspaper I, 5 read both H and I, 10 read both H and T, 7 read both T and I, 4 read all three newspapers. Find the number of people who read at least one of the newspapers?	4	CO3
Q4	Evaluate the following limits. **(a)** $\lim_{x \to 4} \frac{3 - \sqrt{x+5}}{x-4}$ **(b)** $\lim_{x \to 5} \frac{2x^2 - 7x - 15}{x - 5}$	4	CO3
Q5	**(a)** From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are to be there on the committee. In how many ways can it be done? **(b)** In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?	4	CO4

	SECTION B (4Qx10M= 40 Marks)		
Q6	**(a)** Solve $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$ by reducing into the quadratic form. **(b)** If α, β are the roots of $ax^2 + bx + c = 0$, find the equation whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$.	10	CO1
Q7	Solve the following system of equations using Cramer's rule. $x + y + z = 11, 2x - 6y - z = 0, 3x + 4y + 2z = 0$.	10	CO2
Q8	Using the principles of mathematical induction, prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.	10	CO3
Q9	Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second group will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group. **OR** A batch of 40 components contains 5 which are defective. A component is drawn at random from the batch and tested and then a second component is drawn. Determine the probability that neither of the components is defective when drawn **(i)** with replacement **(ii)** without replacement.	10	CO4
	SECTION-C (2Qx20M=40 Marks)		
Q10	**(a)** Evaluate $\int e^{ax} \cos bx \; dx$ using integration by parts technique. **(b)** Find the maxima and minima of the curve $y = x^3 - 3x + 5$ determining the sign of the second derivative.	20	CO3

Q11	**(a)** Solve $2x + 3y - 4z = 26;\ x - 5y - 3z = -87;\ -7x + 2y + 6z = 12$ using the matrix inversion method. **(b)** Determine the values of a and b for which the system $x + 2y + 3z = 6;\ x + 3y + 5z = 9;$ and $2x + 5y + az = b$ has **(i)** no solution **(ii)** unique solution **(iii)** infinite number of solutions. Also, find the infinite number of solutions. **OR** **(a)** Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & -2 & 1 & 6 \end{bmatrix}$ using elementary row transformations. **(b)** Define Orthogonal matrix and find the values of l, m, n and A^{-1} if the matrix $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal.	**20**	**CO2**