Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Matrices **Semester: I**
Program: B.Sc. (Hons.) (Physics/Geology/Chemistry) **Time: 03 hrs.**
Course Code: MATH 1029 G **Max. Marks : 100**

Instructions: Attempt all the questions. Q9 and Q11 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Express the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.	**4**	**CO1**
Q2	Define the Inverse of a square matrix and hence find the inverse of $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$.	**4**	**CO2**
Q3	Define Linear dependency and independency of vectors. Find the condition on "a" for which the set $S = \{(0,1,a),(a,1,0),(1,a,1)\}$ is linearly independent.	**4**	**CO3**
Q4	For the transformation $\xi = x\cos\alpha - y\sin\alpha$; $\eta = x\sin\alpha + y\cos\alpha$, prove that the coefficient matrix A is orthogonal. Hence write the inverse transformation.	**4**	**CO4**
Q5	Find the characteristic polynomial of $A = \begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q6	If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, show that $A(adj\ A) = (adj\ A)A = \|A\|I$.	**10**	**CO1**
Q7	Solve the system $x+y+z=5$; $x+2y+2z=6$; $x+2y+3z=8$ using Crout's decomposition technique.	**10**	**CO3**
Q8	Solve the system $x+2y+3z=5$; $2x+8y+22z=6$; and $3x+22y+82z$ using an appropriate LU decomposition technique.	**10**	**CO3**

Q9	State the Cayley Hamilton Theorem. Verify the Caley Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}. **OR** Define the minimal polynomial of a matrix. If $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$, find its minimal polynomial.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	**(a)** Solve the system $\begin{bmatrix} 2 & -7 & 4 \\ 1 & 9 & -6 \\ -3 & 8 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \\ 6 \end{bmatrix}$ using Gauss-Jordan technique. **(b)** Find the non-trivial solutions of the following system of equations using the concept of rank. $2x + y + 2z = 0$ $x + y + 3z = 0$ $4x + 3y + 8z = 0$	**20**	**CO2**
Q11	Diagonalize the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **OR** Prove that the eigen vectors of $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ are not orthogonal.	**20**	**CO4**