Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2022

Course: Finite Volume Methods **Semester: I**
Program: M. Tech Computational Fluid Dynamics (CFD) **Time: 03 hrs.**
Course Code: ASEG 7044 **Max. Marks: 100**

Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory.

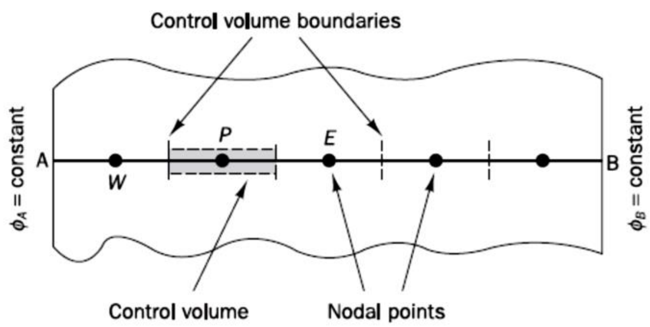
SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Under what conditions, the Navier-Stokes equation will become, • Elliptic • Parabolic • Hyperbolic	**[04]**	**CO2**
Q 2	Which of these models will directly give the conservative equations suitable for the finite volume method? Give brief explanation. a) Finite control volume moving along with the flow b) Finite control volume fixed in space c) Infinitesimally small fluid element moving along with the flow d) Infinitesimally small fluid element fixed in space	**[04]**	**CO1**
Q 3	Which of these terms need a surface integral? Give brief explanation. a) Diffusion and rate of change terms b) Convection and source terms c) Convection and diffusion terms d) Diffusion and source terms	**[04]**	**CO1**
Q 4	For a 1-D convection – diffusion problem, fluid density = 1000 kg/m^3, flow velocity = 1 m/s, diffusion coefficient = 10^{-9} m^2/s, and domain length = 1 m. Will a central difference scheme work, for a numerical solution of this problem (Given that dimension of the solution vector for the TDMA should not exceed 1000)? Give reasons for your answer.	**[04]**	**CO2**
Q 5	Which of these terms need a volume integral while modelling steady flows? Give brief explanation. a) Convection term b) Diffusion term c) Source term d) Rate of change term	**[04]**	**CO3**

<table>
<tr><td colspan="4">SECTION B
(4Qx10M= 40 Marks)</td></tr>
<tr><td>Q 6</td><td>In an incompressible viscous flow, the energy equation is completely decoupled from the continuity and momentum equations, i.e. the solution of energy equation is not required for obtaining pressure and velocity fields. Prove it.</td><td>[10]</td><td>CO2</td></tr>
<tr><td>Q 7</td><td>Consider the steady state diffusion of a property ϕ in a one-dimensional domain defined in figure. The process is governed by

$$\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right)+S=0$$

where Γ is the diffusion coefficient and S is the source term. Boundary values of ϕ at points A and B are prescribed.



Explain the several steps involved in discretizing the geometry and the equation to obtain the appropriate solutions of the governing differential equation.</td><td>[10]</td><td>CO3</td></tr>
<tr><td>Q 8</td><td>Explain with proper example how a pentadiagonal coefficient matrix can be reduced to two sets of tridiagonal coefficient matrix to be solved in sequence.</td><td>[10]</td><td>CO3</td></tr>
<tr><td>Q 9</td><td>Explain the various methods for the approximation of surface integrals over a 2 - dimensional control volume.

OR

Derive the explicit Mac-Cormack time marching algorithm for the solution of transient Euler equations in 2-Dimensions.</td><td>[10]</td><td>CO4</td></tr>
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SECTION-C
(2Qx20M=40 Marks)

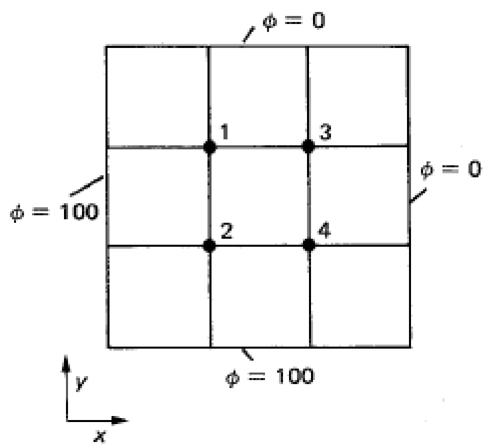
Q 10 | **[20]** | **CO5**

In a steady two-dimensional situation, the variable ϕ is governed by

$$div\ (\rho \boldsymbol{u} \phi) = div(\Gamma grad\phi) + a - b\phi$$

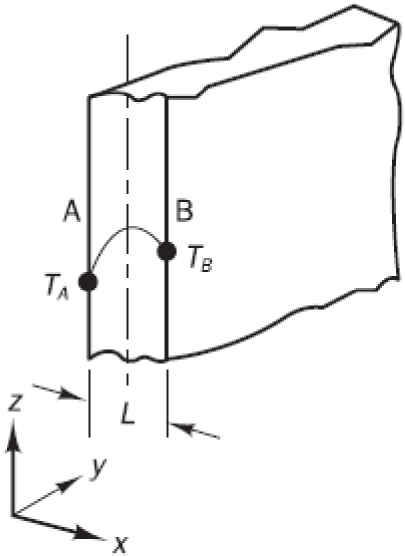
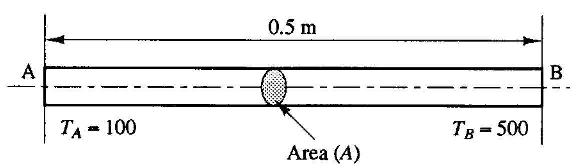
where $\rho = 1$, $\Gamma = 1$, $a = 10$, and $b = 2$. The flow field is such that u = 1 and v = 4 everywhere. For the uniform grid shown in the figure $\Delta x = \Delta y = 1$. The values of ϕ are given for the boundaries. Adopting the control volume design calculate the values of ϕ_1, ϕ_2, ϕ_3, and ϕ_4 by the use of:

(a) The central-difference scheme
(b) The upwind scheme
(c) The hybrid scheme
(d) The power-law scheme



OR

Figure shows a large plate of thickness $L = 2$ *cm* with constant thermal conductivity $k = 0.5$ *W/m.K* and uniform heat generation $q = 1000$ kW/m^3. The faces A and B are at temperatures of 100°C and 200°C respectively. Assuming that the dimensions in the y- and z-directions are so large that temperature gradients are significant in the x-direction only, calculate the steady state temperature distribution. Compare the numerical result with the analytical solution. The governing equation is

	$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right)+q=0$$ 		
Q 11	Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100 °C and 500 °C respectively. The one- dimensional problem sketched in Figure below, is governed by $$\frac{d}{dx}\left(k\,\frac{dT}{dx}\right)=0$$ Calculate the steady state temperature distribution in the rod. Thermal conductivity k equals 1000 W/m/K, cross-sectional area A is 10 x 10^{-3} m^2. Use at least 5 control volumes with appropriate interpolation scheme. 	**[20]**	**CO5**