Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2023

Course: Real Analysis II **Semester: IV**

Program: B.Sc. (H) Mathematics & Int. B.Sc. M.Sc. Mathematics **Time : 03 hrs.**

Course Code: MATH 2051 **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

1) Mention Roll No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Compute by Riemann integration $\int_{-1}^{1} f(x)\,dx$, where $f(x) = x^2$.	**4**	**CO1**
Q 2	Determine the interval of convergence of the power series $\sum\{(1/n)\,(-1)^{n-1}(x)^n\}$.	**4**	**CO3**
Q 3	Give an example to show that the limit of integrals is not equal to the integral of limit.	**4**	**CO2**
Q 4	Find the interval of absolute convergence for the series $\frac{x}{n} + \frac{x^2}{n^2} + \frac{x^3}{n^3} + \cdots$.	**4**	**CO3**
Q 5	Prove that the sequence $\{f_n\}$, where $f_n(x) = nxe^{-nx^2}$, $x \geq 0$ is not uniformly convergent on $[0, k]$, $k > 0$.	**4**	**CO2**

SECTION B

(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Show that the function f defined as follows: $$f(x) = \frac{1}{2^n}, \text{ when } \frac{1}{2^{n+1}} < x < \frac{1}{2^n}, \quad (n = 0, 1, 2, \dots),$$ $$f(0) = 0,$$ is integrable on [0, 1], although it has an infinite number of points of discontinuity.	**10**	**CO1**
Q 7	Test uniform convergence for, the sequence $\{f_n\}$, where $f_n(x) = \frac{\sin nx}{\sqrt{n}}$, for $0 \leq x \leq 2\pi$.	**10**	**CO2**

Q 8	Show by integrating the series for $\frac{1}{(1+x)}$ that if $\lvert x\rvert < 1$, then $\log(1+x) = \sum_{n=1}^{\infty}\left\{{(-1)^{n-1}}/{n}\right\}x^n$.	**10**	**CO3**
Q 9	Find the radius of convergence of the series $\frac{1}{2}x + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$. **OR** Find the radius of convergence of the series $x + \frac{1}{2^2}x^2 + \frac{2!}{3^3}x^3 + \frac{3!}{4^4}x^4 + \cdots$	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**i)** If f and g are integrable on $[a,b]$ and g keeps the same sign over $[a,b]$, then there exists a number μ lying between the bounds of f such that $\int_a^b fg\,dx = \mu\int_a^b g\,dx$. **ii)** If a function is monotonic on $[a,b]$, then it is integrable on $[a,b]$.	**20**	**CO1**
Q 11	**i)** Let f_n be defined by $f_n(x) = 1 - \lvert 1-x^2\rvert^n$, Test the uniform convergence of f_n in the domain $\{x: \lvert 1-x^2\rvert \le 1\} = [-\sqrt{2}, \sqrt{2}]$. **ii)** Let f_n be a sequence of functions defined on an interval I such that $\lim_{n\to\infty} f_n(x) = f(x) \quad \forall\, x \in [a,b]$ and let $M_n = Sup\{\lvert f_n(x) - f(x)\rvert : x \in [a,b]\}$.Then prove that $\langle f_n\rangle$ converge uniformly on [a, b] if $M_n \to 0$ as $n \to \infty$. **OR** **i)** Show that the sequence $\langle f_n\rangle$, where $f_n(x) = nx(1-x)^n$ is not uniformly convergent on closed interval [0, 1]. **ii)** State and prove Cauchy's general principle of uniform convergence.	**20**	**CO2**