Name: **Enrolment No:**	

UPES

End Semester Examination, December 2023

Course: **Differential Calculus** **Semester : I**
Program: **B. Sc. (H) Mathematics** **Time : 03 hrs.**
Course Code: MATH1044 **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 9 and 10 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	If $y = x^2 e^{2x}$, then find the n^{th} derivative of y at $x = 0$.	**4**	**CO1**
Q 2	Find the asymptotes of the curve $$f(x, y) = x^2y^2 - y^2 - 2 = 0$$ which are parallel to the axes.	**4**	**CO2**
Q 3	Compute the value of $\left[\frac{1}{\frac{\partial f}{\partial x}} + \frac{1}{\frac{\partial f}{\partial y}}\right]$ at the point $(1, 2)$ where $f(x, y) = x^3y - xy^3$.	**4**	**CO3**
Q 4	Evaluate $\lim\limits_{(x,y)\to(1,2)} \frac{x^2+7y}{x+y^2}$.	**4**	**CO3**
Q 5	Check whether the following functions $$u(x, y) = \frac{x+y}{1-xy},\ \ v(x, y) = \tan^{-1} x + \tan^{-1} y$$ are functionally dependent or not.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Test the differentiability of the following function at $x = 0$, where $$f(x) = x\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}},\ \text{when } x \neq 0$$ $$= 0 \quad , \quad \text{when } x = 0.$$	**10**	**CO1**
Q 7	Trace the curve $y^2(2a - x) = x^3 (a > 0)$.	**10**	**CO2**
Q 8	State and prove Euler's theorem for partial differentiation of a homogeneous function $f(x, y)$.	**10**	**CO3**

Q 9	Expand $f(x, y) = y^x$ about $(1, 1)$ up to second degree terms and hence evaluate $(1.02)^{1.03}$. **OR** Discuss the maxima and minima of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7.$	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	If $x^x y^y z^z = c$, show that at $x = y = z$, (i) $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ (ii) $\frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \frac{2(x^2-2)}{x(1+\log x)}$. **OR** If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and ϕ is a function of x, y and z, then prove that: $x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z} = u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w}$.	**20**	**CO3**
Q 11	If u, v, w are the roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0,$ in λ, then find the Jacobian of u, v, w with respect to x, y, z.	**20**	**CO4**