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| **Name:**<br>**Enrolment No:** | UPES UNIVERSITY OF TOMORROW |

**UPES**
**End Semester Examination, December 2023**

**Course: Quantum Mechanics and Applications** **Semester: V**
**Program: BSc (H) Physics** **Time : 03 hrs.**
**Course Code: PHYS 3019** **Max. Marks: 100**

**Instructions:** Answers should be clearly marked by drawing a box around them.
There should be a clear separation between problems on the same page.
Use pictures/diagrams in solutions whenever you think is needed.

**SECTION A**
**(5Qx4M=20Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 1 | In a region of space, a particle with mass *m* and with zero energy has a time-independent wave function $\Psi(x) = Axe^{(-x^2/L^2)}$, where *A* and *L* are constants. Determine the potential energy U(x) of the particle. | **4** | **CO2** |
| Q 2 | Use the uncertainty principle to make an order of magnitude estimate for the kinetic energy (in eV) of an electron in a hydrogen atom. | **4** | **CO1** |
| Q 3 | Obtain an expression for the Bohr magneton. | **4** | **CO4** |
| Q 4 | What boundary conditions do valid wave-functions obey? | **4** | **CO1** |
| Q 5 | In a Stern–Gerlach experiment, a collimated beam of neutral atoms is split into 7 equally spaced lines. What is the total angular momentum of the atom? | **4** | **CO3** |

**SECTION B**
**(4Qx10M= 40 Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 6 | What are the possible z components of the vector $\vec{L}$ that represents the orbital angular momentum of a state with *l* = 2? Compute the magnitude (lengt) of the angular momentum. | **10** | **CO4** |
| Q 7 | Show that for a simple harmonic oscillator in the ground state the probability of finding the particle in the classical forbidden region is approximately 16%. | **10** | **CO3** |
| Q 8 | Find the total orbital and spin quantum numbers for carbon atom (Z=6). | **10** | **CO4** |

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| Q 9 | Consider the normal Zeeman effect applied to the 3d to 2p transition. Sketch an energy-level diagram that shows the splitting of the 3d and 2p levels in an external magnetic field. Indicate all possible transitions from each $m_l$ state of the 3d level to each $m_l$ state of the 2p level.<br><br>**Or**<br><br>Compute (Zeeman Effect) the change in wavelength of the 2p → 1s photon when a hydrogen atom is placed in a magnetic field of 2.00 T. | **10** | **CO4** |
| **SECTION-C**<br>**(2Qx20M=40 Marks)** | | | |
| Q 10 | Prove that the operator $L_z$ in the spherical polar coordinate system (r, $\theta$, $\phi$) is represented by $L_z = -i\hbar\frac{\partial}{\partial\phi}$ | **20** | **CO3** |
| Q 11 | State Ehrenfest's theorem. Prove that $\frac{d\langle x\rangle}{dt} = \frac{\langle p_x\rangle}{m}$.<br><br>**Or**<br><br>Consider a simple harmonic oscillator with a Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. Show that $[H,[H,x^2]] = (2\hbar\omega)x^2 - \frac{4\hbar^2}{m}H$. | **20** | **CO2** |

**Standard Physics Constants and their values:**

| Constants | Standard values |
|---|---|
| Planck's constant $(h)$ | $6.626 \times 10^{-34}\ Js$ |
| Speed of light $(c)$ | $3 \times 10^{8}\ m/s$ |
| Boltzmann constant $(k_B)$ | $1.38 \times 10^{-23}\ J/K$ |
| Rest mass of an electron $(m_0)$ | $9.11 \times 10^{-31}\ kg$ or 511 keV/c$^2$ |
| Charge on electron $(e)$ | $1.6 \times 10^{-19}\ C$ |
| Rest mass of a proton $(m_P)$ | $1.67 \times 10^{-27}\ kg$ |