Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, Dec 2023

Course: Integral Equations & Calculus of Variations **Semester: V**

Program: Integrated B.Sc.-M.Sc. Mathematics **Time: 03 hrs.**

Course Code: MATH 3046 **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Suppose $f: [0,1) \to [1, \infty)$ is such that $f(x) = 1 + \int_0^x \left(f(t)\right)^2 dt$. Determine $f(x)$.	**4**	**CO1**
Q2	Determine the cardinality of set S consisting of all the solutions of the integral equation: $$y(x) = e^x + \int_0^1 2xy(t)dt$$	**4**	**CO1**
Q3	The integral equation: $$\phi(x) = \lambda \int_0^1 e^{x+t}.\phi(t)\,dt$$ has a non-trivial solution for some λ. Find such value(s) of λ.	**4**	**CO2**
Q4	Find the resolvent kernel for the Volterra integral equation: $$y(x) = x + \lambda \int_a^x y(t)\,dt$$	**4**	**CO2**
Q5	Find the set $S = \left\{y\left(\left(2n + \frac{1}{2}\right)\pi\right) \middle\vert\ y(x)\ is\ extremal\right\}$ where the variational problem is $I[y(x)] = \int_0^{2\pi}(y^2 - y'^2)dx$; $y(0) = 1, y(2\pi) = 1$.	**4**	**CO3**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Use Laplace transform to determine $y(1)$ from the convolution type integral equation:	**10**	**CO1**

	$$y(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2]\, y(t)\, dt$$		
Q7	Use successive approximation to solve the Fredholm equation: $$u(x) = 1 + \int_0^1 x.u(t)dt$$ with $u_0(x) = 1$ as initial approximation.	**10**	**CO2**
Q8	Determine the smooth function $y(x)$ satisfying $y(0) = y(1) = 1$ that minimizes J where $J[y(x)] = \int_0^1 \left(y'^2 + \frac{4y^2}{x^2}\right) x dx$.	**10**	**CO3**
Q9	If $y_e(x)$ is the extremal of the functional: $$J[y(x)] = \int_0^1 \left(y'^2(x) + 2y(x)\right) dx$$ subject to $y(0) = 0, y(1) = 1$. Find $\inf J[y_e(x)]$.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	State the Isoperimetric problem. Use calculus of variations to find the shortest curve in the first quadrant joining points $P(0,0)$ and $Q(1,0)$ that has area equal to 1 square units beneath it.	**20**	**CO4**
Q11	(a) Show that the integral equation: $$y(x) = f(x) + \lambda \int_0^1 \cos(x+t)\, y(t)dt$$ possesses no solution if $f(x) = x$ but infinitely many solutions if $f(x) = 1$. ***OR*** Determine the eigenvalues and eigenfunctions of the integral equation: $$y(x) = \lambda \int_0^1 \max[(1-x)t, (1-t)x]\, y(t)dt$$ where $0 < x < 1, 0 < t < 1$.	**20**	**CO2**