Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2024

Course: Engineering Mathematics II **Semester: II**
Program: SoAE (All Branches) **Time : 03 hrs.**
Course Code: MATH1051 **Max. Marks: 100**

Instructions: Attempt all questions. There will be internal choice in Q. No. 9 & 11.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	For what values of α and β, the differential equation: $$(\alpha x^2 y - 2\beta y)dx + (2x - x^3)dy = 0$$ becomes exact?	**4**	**CO1**
Q2	Compute the value of $f(1)$ if $$\oint_{\lvert z\rvert=\pi} \frac{1}{(7z-22)} dz = f(z) + \pi$$	**4**	**CO2**
Q3	Use Cauchy's integral formula to evaluate the integral: $$\oint_C \frac{e^{2z}}{(z-1)} dz$$ where C is a circle $z = 1 + 2e^{i2\theta}, \theta \in [0, 2\pi)$ oriented counterclockwise.	**4**	**CO2**
Q4	Determine the image of the unit circle in the complex plane under the linear fractional mapping $f(z) = \frac{2z-1}{z+1}$.	**4**	**CO3**
Q5	Identify the regions S_1, S_2 and S_3 in XY-plane where the partial differential equation $x^2 u_{yy} + y^2 u_{xx} = 0$ is parabolic, hyperbolic and elliptic.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q6	Consider the ordinary differential equation: $$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{5x} - 1$$ Find the solution $y(x)$.	**10**	**CO1**
Q7	Let C be the semi-circular arc of radius unity in the upper-half plane oriented counterclockwise. Evaluate the complex integral: $$\int_C e^{2(\bar{z})^{-1}} dz$$	**10**	**CO2**

Q8	Apply Milne Thomson's method to find an analytic function $f(z)$ such that $Re\{f'(z)\} = 3(x^2 - y^2), f(0) = 1$ and $f(-1) = 0$.	**10**	**CO2**
Q9	Obtain the Taylor's series expansion of the function $f(z) = \sin z + \cos z$ centered at $z = 0$ retaining terms up to degree 3. **OR** Find the Laurent's series expansion of $f(z) = \frac{z}{(z-1)(z+1)}$ in the annular region $0 < \lvert z - 1 \rvert < 1$.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	Consider the function $f(z) = \frac{\sin(z-1)}{(z-1)(e^z + e^{-z})}$. (a) Find the set S of all singularities of $f(z)$. (b) Determine the nature of each singularity in S. (c) Find the residue of $f(z)$ at $z = 1$. (d) Use Cauchy's residue theorem to evaluate $\oint_C f(z)dz$ where C is the circle $\lvert z \rvert = \frac{3}{2}$ oriented counterclockwise.	**20**	**CO3**
Q11	The partial differential equation governing the vibrations in a tightly stretched elastic string of length 2 units between two fixed points is given by: $$4\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}.$$ (a) Use the method of separation of variables to find the most general solution of the above partial differential equation. (b) Impose the conditions $y(0,t) = y(2,t) = 0$ for all $t > 0$ and $y(x,0) = 2\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{3\pi x}{2}\right)$ and $\frac{\partial y}{\partial t}(x,0) = 0$ for all $0 \leq x \leq 2$ on the solution obtained in part (a). **OR** (a) Obtain the partial differential equation corresponding to the family of surfaces $z = f(x^2 - y)$ where f is an arbitrary differentiable function. (b) Find the general solution of partial differential equation: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ (c) Obtain two different particular solutions for the partial differential equation: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = xy$.	**20**	**CO4**