Name: **Enrolment No:**	

UPES

End Semester Examination, May 2024

Course: **Function of Several Variables and PDEs** **Semester : IV**
Program: **B. Sc. (H) Mathematics** **Time : 03 hrs.**
Course Code: MATH2050 **Max. Marks: 100**

Instructions: Attempt all questions from Sections **A, B,** and **C**. Questions **6** and **11** have internal choices**.**

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(0,1,2)$, if $z^3 + xy - y^2 z = 6$.	**4**	**CO1**
Q 2	Define homogeneous function. Check whether the following function $$u(x,y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log_e x - \log_e y}{x^2}$$ is homogeneous or not.	**4**	**CO1**
Q 3	Form partial differential equations by eliminating arbitrary constants a and b from the relation $z = ax + by + a^2 b^2$.	**4**	**CO2**
Q 4	Classify the partial differential equation $5\frac{\partial^2 u}{\partial x^2} - 9\frac{\partial^2 u}{\partial x \partial t} + 4\frac{\partial^2 u}{\partial t^2} = 0$.	**4**	**CO3**
Q 5	Find the solution of the equation $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ using method of separation of variables.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Examine whether the following functions $u(x,y)$ and $v(x,y)$ are functionally dependent or not. If functionally dependent, find the relation between them. $$u(x,y) = \frac{x-y}{x+y},\ v(x,y) = \frac{x+y}{x}.$$ **OR** Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.	**10**	**CO1**
Q 7	Solve the first-order partial differential equation $$xy^2 p + y^3 q = (zxy^2 - 4x^3),$$ by using Lagrange's method.	**10**	**CO2**

Q 8	Apply Charpit's method to find the complete solution of the non-linear partial differential equation $z^2 = pqxy$ (where $p \equiv \frac{\partial z}{\partial x}$ and $q \equiv \frac{\partial z}{\partial y}$.)	**10**	**CO3**
Q 9	Find the temperature in a laterally insulated bar of $2\ cm$ length whose ends are kept at zero temperature and the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**(i)** Solve the partial differential equation $\left(4D^2 + 12DD' + 9D'^2\right)z = e^{3x-2y}$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. **(ii)** Reduce the differential equation $3\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.	**10+10**	**CO3**
Q 11	A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = \text{A}\sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = A\sin(\pi x/l)\cos(\pi ct/l)$. **OR** A tightly stretched string with fixed end points $x = 0$ and $x = \pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.03\sin x - 0.04\sin 3x$, then find the displacement $y(x,t)$ at any point of string at any time t.	**20**	**CO4**